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TANGENTIALLY AFFINE FOLIATIONS AND LEAFWISE AFFINE FUNCTIONS ON THE TORUS

In memory of Professor Itiro Tamura

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1. Introduction

Let $\mathcal{F}$ be a foliation on a manifold M. We say that $\mathcal{F}$ is *tangentially affine* if M is covered by a collection of $\mathcal{F}$-distinguished charts for which the coordinate transformations are affine in the direction tangent to $\mathcal{F}$. This notion is, in a sense, dual to that of *transversely affine* foliation ([In]). Tangentially affine foliations appear in several branches of mathematics: for example, a Lagrangian foliation on a symplectic manifold is tangentially affine (See [AN]), and a supermanifold (in the sense of Rogers [Ro]) over a finite dimensional Grassmann algebra has a family of tangentially affine foliations ([BG], [RC1], [RC2], [CRT]).

The following problems naturally arise: (1) Which foliation admits a tangentially affine structure? (2) Given a tangentially affine foliation $\mathcal{F}$ on a compact manifold M, does there exist a leafwise affine function on M which is nonconstant along leaves of $\mathcal{F}$? And if so, how many? Problem (1) is studied in [Fu] under additional condition that all leaves are affinely complete. As for (2), the authors cannot find any positive answer in the literature.

The purpose of this paper is to give complete answers to these problems for the 2-torus T^2. The results are as follows.

THEOREM 1. *Every codimension one smooth foliation on T^2 admits a tangentially affine structure.*

Example. There exists a tangentially affine foliation on T^2 admitting a leafwise affine function which is nonconstant along leaves.

THEOREM 2. *Let $\mathcal{F}$ be a tangentially affine foliation on T^2 and F a leafwise affine function on T^2 for $\mathcal{F}$. Then F is uniquely determined by the values on the union of compact leaves of $\mathcal{F}$.*

THEOREM 3. *Let $\mathcal{F}$ be a tangentially affine foliation on T^2 and L a compact*

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