

## ASYMPTOTIC BEHAVIOR OF ALMOST-ORBITS OF NONEXPANSIVE SEMIGROUPS WITHOUT CONVEXITY

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### Abstract

We first prove a result on the asymptotic behavior of almost-orbits of nonexpansive semigroups without convexity in a Hilbert space. This is a generalization of results of Rodé [7] and Takahashi [10]. Further we prove a fixed point theorem for Lipschitzian semigroups without convexity. This is a generalization of results of Lau [3], Takahashi [8], [10] and Ishihara [2].

**1. Introduction.** Let  $H$  be a real Hilbert space with norm  $\|\cdot\|$  and inner product  $(\cdot, \cdot)$  and let  $C$  be a nonempty subset of  $H$ . A mapping  $T: C \rightarrow C$  is said to be Lipschitzian if there exists a nonnegative number  $k$  such that

$$\|Tx - Ty\| \leq k\|x - y\| \quad \text{for every } x, y \in C$$

and nonexpansive in the case of  $k=1$ . Let  $S$  be a semitopological semigroup, i.e., a semigroup with a Hausdorff topology such that for each  $s \in S$ , the mappings  $t \rightarrow t \cdot s$  and  $t \rightarrow s \cdot t$  of  $S$  into itself are continuous. Then a family  $\mathcal{S} = \{T_s : s \in S\}$  of mappings of  $C$  into itself is called a Lipschitzian semigroup on  $C$  if it satisfies the following:

(1)  $T_{st}x = T_s T_t x$  for all  $s, t \in S$  and  $x \in C$ ;

(2) for each  $x \in C$ , the mapping  $s \rightarrow T_s x$  is continuous on  $S$ ;

(3) for each  $s \in S$ ,  $T_s$  is a Lipschitzian mapping of  $C$  into itself with Lipschitz constant  $k_s$ . A Lipschitzian semigroup  $\mathcal{S} = \{T_t : t \in S\}$  on  $C$  is said to be nonexpansive if  $k_s = 1$  for every  $s \in S$ . Recently, Takahashi [10] proved a nonlinear ergodic theorem and a fixed point theorem for nonexpansive semigroups without convexity in a Hilbert space. On the other hand, Miyadera-Kobayasi [4] introduced the notion of an almost-orbit of nonexpansive semigroups and established the weak and strong almost convergence of such an almost-orbit; see also [1], [11], [12].

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