M. JAMBU
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ANALYSIS AND TOPOLOGY OF HYPERPLANE COMPLEMENTS: THE GENERALIZED WITT FORMULA

BY MICHEL JAMBU

Introduction.

The *classical Witt formula* which gives the dimensions of the homogeneous components of the free Lie algebra over a finite set, has a nice interpretation as a relation between the topology, i.e. cohomology and homotopy of the complement of a finite set of $\boldsymbol{C}$, and the analysis, i.e. an ordinary linear differential equation with regular singular points at this finite set of $\boldsymbol{C}$.

Such a relation remains true for complements of some hyperplane arrangements such as *complexified Coxeter arrangements* and *fiber-type arrangements*.

Namely, let $\mathcal{A}$ be a finite family of hyperplanes of $\boldsymbol{C}^n$ through the origin and let $M=\boldsymbol{C}^n \setminus \bigcup_{H\in A} H$ be the complement. The cohomology algebra $H^*(M; K)$, where $K=\boldsymbol{Z}, \boldsymbol{Q}, \boldsymbol{R}$ or $\boldsymbol{C}$ is isomorphic to $\mathcal{E}/I$ where $\mathcal{E}$ is the free exterior algebra over $\mathcal{A}$ and I is the ideal defined by some dependence relations between the hyperplanes of $\mathcal{A}$. Moreover:

$$P_M(t)=\sum_{p\geq 0}(\operatorname{rank} H^p(M))t^p=\sum_{x\in L(A)}\mu(x)(-t)^{\operatorname{codim} x}$$

where $L(\mathcal{A})$ is the lattice of intersections hyperplanes ordered by reverse inclusion, $\mu(x)=\mu(0, x)$, μ being the Möbius function. These results are due to P. Orlik and L. Solomon [OS].

The algebra of the integrable logarithmic connections along $\mathcal{A}$ is called the *holonomy Lie algebra* of M and is denoted $\mathcal{G}_M$. T. Kohno [K1] showed that $\mathcal{G}_M=\operatorname{Lib}(A)/\mathcal{N}$ where $|A|=|\mathcal{A}|$ and $\mathcal{N}$ is the ideal defined by some dependence relations between the hyperplanes of $\mathcal{A}$.

Let $\mathcal{L}_M$ be the *Malcev algebra* of M which is obtained (cf Sullivan [S]) from the 1-minimal model of M. Using the mixed Hodge structure on the minimal model, T. Kohno [K2] showed that:

$$\mathcal{G}_M^* \approx \mathcal{L}_M$$

where $\mathcal{G}_M^*$ is the nilpotent completion of $\mathcal{G}_M$.

Then T. Kohno [K3] proved that:

$$\varphi_j(M)=\dim(\Gamma_j\mathcal{G}_M/\Gamma_{j+1}\mathcal{G}_M)=\operatorname{rank}(\Gamma_j\pi_1(M)/\Gamma_{j+1}\pi_1(M))$$

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