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KILLING FIELDS PRESERVING TOTALLY GEODESIC, CODIMENSION-ONE FOLIATIONS

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§ 1. Introduction

Let M be a complete manifold, endowed with a codimension-one foliation $\mathcal{F}$. We want to study the Lie algebra $\mathcal{G}$ of Killing fields preserving the foliation (i.e., Killing fields such that the isometries of their one-parameter group send leaves of $\mathcal{F}$ onto leaves of $\mathcal{F}$).

In [5], Johnson and Whitt proved that when the foliation is totally geodesic (i.e., leaves are totally geodesic submanifolds) and all the leaves are compact, then any Killing field preserves $\mathcal{F}$. Later, Oshikiri (see [7]) proved the same result for the case when the manifold is compact and $\mathcal{F}$ is totally geodesic. Nevertheless, in the general case all Killing fields do not preserve foliations. For example, in the euclidean plane foliated by lines parallel to the 0X-axis, Killing fields associated to rotations do not preserve the foliation.

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§ 2. Totally geodesic foliations

First of all, let us recall that any codimension-one foliation which admits an orthogonal Killing field must be totally geodesic (see [3] for instance). For this reason, from now on we shall only consider totally geodesic foliations. The universal cover of a manifold with such a structure verifies the following

THEOREM 1. (see [2]) *Let $(M, \mathcal{F})$ be a complete manifold with a codimension-one, totally geodesic foliation. Let $\tilde{M}$ be the universal cover of M. Then $\tilde{M}$ is trivially foliated as $\tilde{L}\times\boldsymbol{R}$, where $\tilde{L}$ is the universal cover of any leaf and the induced metric reads $ds^2_{\tilde{M}}=ds^2_{\tilde{L}}+f^2d\tilde{t}^2$, where $f:\tilde{M}\to(0,\infty)$ is a C^∞ function.*

In order to simplify calculations, it will be convenient to give a characterization of Killing fields preserving foliations. Let $(M, \mathcal{F})$ be a complete manifold with a codimension-one, totally geodesic foliation. With the notations of Theorem 1,

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