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NULL FINITE TYPE HYPERSURFACES IN SPACE FORMS

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0. Introduction

In [5], Chen gives a classification of null 2-type surfaces in the Euclidean 3-space and he shows in [6] that a similar characterization cannot be given for a surface in the Euclidean 4-space. In fact, helical cylinders in Euclidean 4-space are characterized as those surfaces of null 2-type and constant mean curvature.

In this paper we give a characterization of null 2-type hypersurfaces in a space of constant sectional curvature $\overline{M}^{n+1}(k)$ and an approach to hypersurfaces of null 3-type. Indeed, we get a generalization of Chen's paper [5] not only by considering hypersurfaces, but also taking them in space forms.

In spherical and hyperbolic cases we show that there is no null 2-type hypersurface, so that the Euclidean case becomes the most attractive situation where our classification works on. Actually, we show that Euclidean hypersurfaces of null 2-type and having at most two distinct principal curvatures are locally isometric to a generalized cylinder. Why the hypothesis on principal curvatures? First, we think this is the most natural one, because, after Chen's paper, we know that cylinders are the only surfaces of null 2-type in Euclidean 3-space. Secondly, it is well-known that a Euclidean isoparametric hypersurface has at most two distinct principal curvatures, so that if it has exactly two, then one of them has to be zero. Our classification depends strongly on that isoparametricity condition. Finally, bounding the number of principal curvatures is not as restrictive as one could hope. As a matter of fact, the families of conformally flat and rotational hypersurfaces satisfy that hypothesis and both are sufficiently large so that it is worth trying to give a characterization of some subfamily of them in order to get along in their classifications. To this effect, we characterize rotational and conformally flat hypersurfaces of null 2-type.

As for hypersurfaces of null 3-type one immediately sees that they are not difficult to handle when they have constant mean curvature, because a nice

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