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GENERALIZED HOPF MANIFOLDS, LOCALLY CONFORMAL KAEHLER STRUCTURES AND REAL HYPERSURFACES

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Abstract

We study the geometry of submanifolds of complex Hopf manifolds endowed with the (locally conformal Kaehler) Boothby metric.

1. Generalized Hopf manifolds and the Boothby metric.

Let $a\in \boldsymbol{C}$, $0<|a|<1$, be a fixed complex number; let G_a be the discrete group of complex analytic transformations of $W=\boldsymbol{C}^n-\{0\}$, $n>1$, generated by $z\mapsto az$, $z\in W$. Then G_a acts freely and properly discontinuously on W, see [28], vol. II, p. 137, so that the quotient space $\boldsymbol{H}_a^n=W/G_a$ becomes in a natural way a complex n-dimensional manifold. This is the well known *complex Hopf manifold*. In their attempt to construct complex structures on products $S^1\times L$, where S^1 is the unit circle and L an odd dimensional homotopy sphere, E. Brieskorn & A. Van de Ven, [3], have generalized Hopf manifolds a follows. Let $n\geqq 1$ and $(b_0, \cdots, b_n)\in \boldsymbol{Z}^{n+1}$, $b_j\geqq 1$, $0\leqq j\leqq n$. Let $(z_0, \cdots, z_n)$ be the natural complex coordinates on $\boldsymbol{C}^{n+1}$. Define $X^{2n}(b)=X^{2n}(b_0, \cdots, b_n)\subset \boldsymbol{C}^{n+1}$ by the equation:

$$(z_0)^{b_0}+\cdots+(z_n)^{b_n}=0$$

Then $X^{2n}(b)$ ia an affine algebraic variety with one singular point at the origin of $\boldsymbol{C}^{n+1}$ if $b_j\geqq 2$, $j=0, \cdots n$ (and without singularities if $b_j=1$ for at least one j). Next $B^{2n}(b)=X^{2n}(b)-\{0\}$ is a complex n-dimensional manifold, referred hereafter as the *Brieskorn manifold* determined by the integers $b_0, \cdots, b_n$. See [2]. There is a natural holomorphic action of $\boldsymbol{C}$ on $\boldsymbol{B}^{2n}(b)$ given by:

$$t(z_0, \cdots, z_n)=\left(z_0 \exp\left(-\frac{tw_a}{b_0}\right), \cdots, z_n \exp\left(-\frac{tw_a}{b_n}\right)\right) \tag{1}$$

where $t\in \boldsymbol{C}$, $w_a=-\log|a|-i\Phi_a$, $\Phi_a=\arctan(Im(a)/Re(a))$, $-\pi/2<\Phi_a<\pi/2$,

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