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KODAI MATH. J.
14 (1991), 358—365

DUAL CONVERGENCE THEOREMS FOR THE INFINITE PRODUCTS OF RESOLVENTS IN BANACH SPACES

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1. Introduction

Let E be a Banach space, $A \subset E \times E$ an m-accretive operator, and J_r the resolvent of A. Given a sequence $\{r_n\}_{n=0}^{\infty}$ of positive reals and $x_0 \in E$, we define an iterative scheme by

$$x_{n+1} = J_{r_n} x_n, \qquad n=0, 1, 2, \cdots . \tag{1}$$

We shall consider this scheme in particular under the assumption that

$$\sum_{n=0}^{\infty} r_n = \infty . \tag{2}$$

The convergence of (1) in Hilbert spaces has been studied by Rockafellar [17], Brézis and Lions [2], and Pazy [11]. Bruck and Reich [4] and Reich [14] have obtained several results in uniformly convex Banach spaces. Bruck and Passty [3] have established the convergence of weighted averages $y_n = \sum_{i=0}^{n} r_i x_i / \sum_{i=0}^{n} r_i$ in the same Banach space.

The purpose of this paper is to study convergence theorems for iterative scheme (1) in Banach spaces. In Section 3. we prove a dual convergence theorem (Theorem 1) for (1) in a reflexive and strictly convex Banach space with a uniformly Gâteaux differentiable norm, and then apply this result to study the problem of weak convergence. We also use Theorem 1 to show a result in a Hilbert space, which is closely related to the results of Brézis and Lions [2], and Pazy [11]. In Section 4, we present additional results. Furthermore, using the method of the proof of Theorem 1, we give a related result on the asymptotic behavior of a certain nonlinear evolution equation.

2. Preliminaries

Let E be a real Banach space and let I denote the identity operator. Re-

* This paper was studied during stay at Tokyo Institute of Technology under the financial support by Korea Science and Engineering Foundation, 1990.

Received December 25, 1990.