B.Y. CHEN
KODAI MATH. J.
14 (1991), 341—349

LINEARLY INDEPENDENT, ORTHOGONAL AND EQUIVARIANT IMMERSIONS

BY BANG-YEN CHEN

Abstract

In this article we define the notions of linearly independent and orthogonal immersions and introduce the notion of adjoint hyperquadrics of linearly independent immersions. We investigate the relations between linearly indedendent immersions, orthogonal immersions, equivariant immersions and adjoint hyperquadrics. Several results in this respect are obtained.

1. Introduction

Let $x: M \to E^m$ be an immersion from an n-dimensional, connected manifold M into the Euclidean m-space E^m. With respect to the Riemannian metric g on M induced from the Euclidean metric of the ambient space E^m, M is a Riemannian manifold. Denote by Δ the Laplacian operator of the Riemannian manifold (M, g). The immersion x is said to be *of finite type* (cf. [1, 2] for details) if each component of the position vector field of M in E^m, also denoted by x, can be written as a finite sum of eigenfunctions of the Laplacian operator, that is, if

$$x = c + x_1 + x_2 + \cdots + x_k \tag{1.1}$$

where c is a constant vector, $x_1, \cdots, x_k$ are non-constant maps satisfying $\Delta x_i = \lambda_i x_i$, $i = 1, \cdots, k$. If in particular all eigenvalues $\{\lambda_1, \cdots, \lambda_k\}$ are mutually different, then the immersion x (or the submanifold M) is said to be of *k-type* and the decomposition (1.1) is called the *spectral decomposition* of the immersion x.

For a finite type immersion whose spectral decomposition is given by (1.1) we shall always assume in this article that the eigenvalues $\{\lambda_1, \cdots, \lambda_k\}$ satisfy $\lambda_1 < \cdots < \lambda_k$ for simplicity. Moreover, for a such immersion we shall choose the Euclidean coordinate system $(u_1, \cdots, u_m)$ on E^m in such way that c is its origin. Therefore, with respect to the Euclidean coordinate system so chosen, the spectral decomposition of x is given by

Received November 13, 1990.