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1-TYPE SUBMANIFOLDS OF THE COMPLEX PROJECTIVE SPACE

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§ 0. Introduction.

The standard way to define the complex projective space CP^m is by means of Hopf fibration $\pi: S^{2m+1} \to CP^m$ where CP^m is obtained as a quotient space of $S^{2m+1} \subset C^{m+1}$ under the natural action of the group S^1 of complex numbers of norm 1. It is not completely trivial to find an isometric embedding of CP^m into a Euclidean space, the most natural one (the first standard embedding) defined as follows. For $p \in CP^m$ pick $z \in \pi^{-1}(p) \subset S^{2m+1}$ and let $\phi(p) = z\bar{z}^t$ (z is regarded as a column vector in C^{m+1}). Then ϕ is a well defined map that embeds CP^m into the set $H(m+1)$ of Hermitian matrices of degree $m+1$, the latter being a Euclidean space of dimension $N=(m+1)^2$. Now if $x: M^n \to CP^m$ is an isometric immersion of a compact n-dimensional Riemannian manifold into the complex projective space, than we also have the associated immersion $\tilde{x} = \phi \circ x : M^n \to H(m+1) = E^N$.

On the other hand, there is a notion of finite type immersion $f: M^n \to E^N$ whereby a compact Riemannian manifold M^n is said to be of k-type (via f) if f is globally decomposable as $f = f_0 + f_1 + \cdots + f_k$, where $f_0 = \text{const}$ (could be 0) and f_i's are vector eigenfunctions of Laplacian from $k+1$ different eigenspaces [3]. For example, a well known result of Takahashi [15] characterizes compact 1-type submanifolds as minimal in a hypersphere of E^N. Studying finite type immersions is difficult in general, but there have been several results on low type submanifolds of CP^m. In particular, A. Ros has the following classification of CR-minimal submanifolds of CP^m which are of 1-type via the first standard embedding.

THEOREM A [10]. *Let M^n be a compact CR-minimal submanifold of CP^m. Then M^n is of 1-type via ϕ if and only if*

a) *n is even and M^n is congruent to the complex projective space $CP^{n/2}$ immersed as a totally geodesic complex submanifold of CP^m.*

b) *M^n is a totally real minimal submanifold of a complex totally geodesic CP^n in CP^m.*

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