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NOTE ON HECKE OPERATORS AND COHOMOLOGY OF $PSL_2(Z)$

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Introduction.

In this note we study the action of Hecke operators on 1-dimensional cohomology group of the modular group $G=PSL_2(Z)$ with the coefficient module W, the even degree parts of the polynomial algebra $Z[x, y]$, or its reduction modulo a prime power l, $W/l=(Z/lZ)[x, y]$. The cohomology group $H^n(G; W/l)$ is a module over $H^0(G; W/l)=(W/l)^G$, the invariants of W/l. The ring $(W/l)^G$ is known by Dickson [1]. We notice the relation between the above module structure and the action of Hecke operators. Then we obtain some congruences for the eigenvalues of Hecke operators on modular forms.

THEOREM. *Let λ_l bn the eigenvalue of the Hecke operator T_l in $M_k{}^0(G)$; the set of all cusp forms of weight k. Then*

(1) $\lambda_7\equiv 0 \bmod 7$ *if* $k\equiv 10, 14 \bmod 42$

(2) $\lambda_{11}\equiv 0 \bmod 11$ *if* $k\equiv 14 \bmod 110$.

where $\lambda_l\equiv 0 \bmod l$ means λ_l/l is an algebraic integer.

The above results are largely extended in E. Papier's up coming paper [5]. Papier kindly corrected many errors in the first version of this note. She also suggested Proposition 4.3. The authors wish to thank her heartly. They also thank to S. Mizumoto for conversations and suggestion, in particular the computation (1.6) is due to him.

§ 1. Hecke operators and the Eichler-Shimura isomorphism.

Let $G=PSL_2(Z)$ be the modular group and $V=Z[x, y]$, $|x|=|y|=1$, be the polynomial algebra over Z. If we denote the positive even degree parts of V by W. Then G acts on W by $gP(x, y)=P((x, y)g)$ for $g\in G$ and $P(x, y)\in W$. For any G-module E, the Eichler cohomology group $H_P^1(G; E)$ is defined to be the kernel of the restriction map $j^*: H^1(G; E)\to H^1(G_\infty; E)$, here G_∞ denotes

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