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LOCAL MAXIMA OF THE SPHERICAL DERIVATIVE

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Abstract

Let a function f be nonconstant and meromorphic in a domain D in the plane, and let $M(f)$ be the set of points where the spherical derivative $|f'|/(1+|f|^2)$ has local maxima. The components of $M(f)$ are at most countable and each component is (i) an isolated point, (ii) a noncompact simple analytic arc terminating nowhere in D, or, (iii) an analytic Jordan curve. Tangents to a component of type (ii) or (iii) are expressed by the argument of the Schwarzian derivative of f. If Δ is the Jordan domain bounded by a component of type (iii) and if $\Delta \subset D$, then the spherical area of the Riemann surface $f(\Delta)$ can be expressed by the total number of the zeros and poles of f' in Δ. Solutions of a nonlinear partial differential equation will be considered in connection with the spherical derivative.

1. Introduction.

Let f be a nonconstant meromorphic function in a domain D in the complex plane $\boldsymbol{C}=\{|z|<+\infty\}$. The spherical derivative of f at $z \in D$ is defined by

$$f^{\#}(z)=\begin{cases} |f'(z)|/(1+|f(z)|^2) & \text{if } f(z)\neq\infty; \\ |(1/f)'(z)| & \text{if } f(z)=\infty. \end{cases}$$

We let $M(f)$ be the set of points $z \in D$ where $f^{\#}$ has local maxima, namely, $f^{\#}(z) \geqq f^{\#}(w)$ in $\{|w-z|<\delta\} \subset D$ for $\delta>0$ depending on f and z.

The purpose of the present paper is to investigate $M(f)$ in detail. We begin with a classification.

THEOREM 1. *Let f be nonconstant and meromorphic in a domain $D \subset \boldsymbol{C}$ with nonempty $M(f)$. Then, the connected components of $M(f)$ are at most countable and each component is one of the following*:

(I) *An isolated point.*

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