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SELF MAPS OF $\Sigma^k CP^3$ FOR $k \geqq 1$

Dedicated to Prof. Shôro Araki on his 60th birthday

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§ 1. Introduction.

Throughout this note, all spaces, maps and homotopies are assumed to be based, and we will not distinguish the map and its homotopy class.

For two topological spaces X and Y, we denote by $[X, Y]$ the set of homotopy classes of maps from X to Y.

If $X=Y$, then the set $[X, X]$ becomes a monoid with its multiplication induced from the composition of maps and we put $M(X)=[X, X]$.

Let $\mathcal{E}(X)$ be the group consisting of all invertible elements of $M(X)$ and we call it the group of self-homotopy equivalences of X.

The group $\mathcal{E}(X)$ has been studied by several authors since the paper of W. D. Barcus and M. G. Barratt [1] appeared.

However, we have not yet obtained an effective method for calculating it except classical ones, and its structure also has not been clarified sufficiently. Furthermore, very little is known about it even when X is a simply connected CW complex with three cells which is not a H-space.

Then the purpose of this note is to study the multiplicative structure of $M(\Sigma^k CP^3)$ and determine the group $\mathcal{E}(\Sigma^k CP^3)$ for $k \geqq 1$, where CP^n is the complex n dimensional projective space and Σ^k denotes the k-times iterated suspension.

We denote by Z_n (resp. Z/n) the multiplicative (resp. additive) cyclic group of order n.

Our main results are stated as follows:

THEOREM A. (*The case* $k=1$)

(1) *There is an exact sequence*

$$0 \longrightarrow Z \xrightarrow{\nabla} \mathcal{E}(\Sigma CP^3) \xrightarrow{\Sigma} Z_2 \times Z_2 \times Z_2 \longrightarrow 1.$$

(2) $\mathcal{E}(\Sigma CP^3)=Z \ltimes (Z_2 \times Z_2)$ (*semidirect product*).

Next we consider the case $k \geqq 2$.

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