T. TADA
KODAI MATH. J.
14 (1991), 134—143

ESSENTIAL SETS OF PICARD PRINCIPLE FOR ROTATION FREE DENSITIES

Dedicated to Professor Masanori Kishi on his 60th birthday

BY TOSHIMASA TADA

We denote by Ω the punctured unit disk $0<|z|<1$ and consider a Schrödinger equation

$$(1) \qquad (-\Delta+P(z))u(z)=0 \quad \left(\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2},\ z=x+yi\right)$$

on Ω. The potential P is assumed to be nonnegative and locally Hölder continuous on $0<|z|\leqq 1$ and referred to as a *density* on Ω. We say that the *Picard principle* is valid for P (at the origin $z=0$) if the set $F_P(\Omega)$ of nonnegative solutions of (1) on Ω with vanishing boundary values on the unit circle Γ: $|z|=1$ is generated by one element u of $F_P(\Omega)$: $F_P(\Omega)=\{cu : c\geqq 0\}$. In other words the Picard principle is valid for P at the origin if and only if the Martin ideal boundary of Ω over the origin with respect to (1) consists of one point. Let P be a density on Ω for which the Picard principle is valid and Q a density on Ω with $Q\leqq P$ on Ω. The Picard principle for Q is generally invalid ([8], [9]). However the Picard principle for Q is valid if densities P and Q are *rotation free*, i.e. $P(z)=P(|z|)$ and $Q(z)=Q(|z|)$ on Ω ([7]). Moreover the Picard principle for Q is valid if $Q\leqq P$ on a subset of Ω for some densities P ([2]). In this note we will study this subset of Ω for the special densities $P(z)=|z|^{-2}$ and $P(z)=(\log|z|)^2/|z|^2$.

Hereafter *every* density P on Ω in consideration is assumed to be rotation free and is mainly viewed as a function $P(r)$ of r in the interval $(0, 1]$. In order to define the above subsets of Ω we take two sequences $\{a_n\}_1^\infty$, $\{b_n\}_1^\infty$ which are always supposed to satisfy

$$0<b_{n+1}<a_n<b_n<1 \quad (n=1, 2, \cdots), \qquad \lim_{n\to\infty} a_n=0$$

and we set

$$A=A(\{a_n\}, \{b_n\})=\bigcup_{n=1}^{\infty}[a_n, b_n].$$

This work is partially supported by Grant-in-Aid for Scientific Research (No. 62302003), Ministry of Education, Science and Culture.

This work is completed while the author is engaged in the research at Department of Electrical and Computer Engineering, Nagoya Institute of Technology.

Received May 31, 1989; Revised September 17, 1990.