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REAL ZEROS OF SOLUTIONS OF SECOND ORDER LINEAR DIFFERENTIAL EQUATIONS

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1. Introduction

We consider the second order linear differential equation

$$f''+Af=0, \tag{1.1}$$

where A is an entire function. For an entire function f, let $\rho(f)$ be its order, $\mu(f)$ its lower order, $\lambda(f)$ the exponent of convergence of its zeros and $\lambda_{NR}(f)$ the exponent of convergence of its non-real zeros. In addition, we assume that the reader is familiar with the standard notation of Nevanlinna theory (see [4]).

When A is a polynomial, the distribution of zeros of solutions of (1.1) has been studied extensively. The following theorem is well-known ([1]).

THEOREM A. *If A is a polynomial of degree $n\geqq 1$, then every solution $f\not\equiv 0$ of* (1.1) *satisfies*

$$\rho(f)=(n+2)/2, \tag{1.2}$$

and if f_1, f_2 are two linearly independent solutions of (1.1), *then*

$$\lambda(f_1f_2)=(n+2)/2. \tag{1.3}$$

Furthermore, G. Gundersen proved the following ([2]).

THEOREM B. *Under the hypothesis of Theorem A,*

$$\lambda_{NR}(f_1f_2)=(n+2)/2. \tag{1.4}$$

When A is transcendental, we apply the lemma on the logarithmic derivative in Nevanlinna theory to (1.1) and can easily deduce that any solution $f\not\equiv 0$ of (1.1) satisfies

$$\rho(f)=+\infty. \tag{1.5}$$

By analogy with Theorem A and Theorem B, we may hope that

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