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FOUNDATIONS OF CALCULUS ON SUPER EUCLIDEAN SPACE $\mathfrak{R}^{m|n}$ BASED ON A FRÉCHET-GRASSMANN ALGEBRA

Dedicated to Professor T. Kimura on the occasion of his 60th birthday

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Abstract.

We define a Fréchet-Grassmann algebra with infinitely many generators as the supernumber algebra. Using this, we define a so-called super Euclidean space and may develop elementary analysis on it. In doing this, we clarify the relation between Grassmann generators and odd variables. Moreover, we construct a certain Hamilton flow on the super Euclidean space, corresponding to the 'classical' orbit of the Pauli equation, for which we define the action integral, van Vleck determinant etc. as similar as we do on the Euclidean space.

Introduction

After the pioneering works of Martin [20, 21] in 1959, who considered a generalization of the classical mechanics on a ring with arbitrary generators, Berezin started independently his endeavor of a generalization of analysis in which the Grassmann variables would play a part on equal footing with real variables. (One may find more general idea in Manin [19] where he claimed that there should be at least 'three dimensions = ordinary, odd and arithmetic dimensions' in geometry.) There are many works by Berezin, but seemingly he did not distinguish the Grassmann generators and the (odd) variables because he considered his supermanifold rather sheaf theoretically. Roughly speaking, for an (ordinary) C^∞-manifold X of $\dim X=m$, he considered a ringed space $(X, \mathcal{A}(X))$ as his supermanifold of dimension $\mathcal{A}(X)=C^\infty(X)\otimes\Lambda(R^n)$. See, his book edited by Kirillov [2] and Leites [17].

Supersymmetric theory is now widely used by physicists, and the need of an infinite number of generators is recognized by some of them especially when they want to 'quantize classical systems'. Therefore, there are many trials to define the 'supernumber' based on the Grassmann algebra with infinitely many generators. For example, Rogers [23] introduced a Banach-Grassmann algebra

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