R. MIYAOKA
KODAI MATH. J.
14 (1991), 13—41

LIE CONTACT STRUCTURES AND NORMAL CARTAN CONNECTIONS

Dedicated to Professor Noboru Tanaka on his 60th birthday

BY REIKO MIYAOKA

§ 0. Introduction.

Studies of G-structures and connections concerned with differential systems on a manifold have been deeply developed by N. Tanaka in these ten years [T1, 2, 3, …].

Classical examples of G-structures are the projective and the conformal structures on a differentiable manifold. The former is a geometry with model space $P^n(\boldsymbol{R})$ and group $PL(n, \boldsymbol{R})$, [KN], the latter is with model space S^n, the Möbius space, and group $PO(n+1, 1)$, [O].

Here, we study *a Lie contact structure* considered by H. Sato [S, SY], which is a geometry over $(2n-1)$-dimensional contact manifolds with model space T_1S^n, the unit tangent bundle of the n-dimensional standard sphere, and group $PO(n+1, 2)$. Since the grading of the Lie algebra of $O(n+1, 2)$ is from -2 to 2 (of the second kind), the structure is much more complicated than the classical ones.

In this paper, we give basic facts on Lie contact structures in § 1. In § 2, we discuss on some examples. In particular, the structure given on the unit tangent bundle T_1M of a riemannian manifold M is important because it is related with both conformal structure of M and contact structure of T_1M [M2]. To investigate the relation, we must calculate the curvature of the normal Cartan connection defined in § 3. In fact, by the connection theory due to N. Tanaka [T1, 2, 3], which is thoroughly applicable to Lie contact structures, the normal Cartan connection determines the structure completely (§ 4). Nevertheless, the concrete description of its torsion and curvature have not yet been done. In § 5, we give an explicit description of these objects, proving at the same time the existence of the connection in a constructive way. All of these results are used in [M2] to calculate the curvature of the normal Cartan connection of the Lie contact structure on T_1M. Note that the definition of normal Cartan connections in [T2] is different from the one in [T3], the latter would be preferable theoretically. We adopt here the definition in [T3].

Received April 19, 1989; Revised April 26, 1990.