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A NOTE ON CARATHÉODORY AND KOBAYASHI PSEUDODISTANCES

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Introduction.

Recently, Jarnicki and Pflug [7], [8] presented an effective formula for the Carathéodory pseudodistance from the origin on logarithmically coned, complete Reinhardt domains in $\boldsymbol{C}^n$. The aim of this note is to establish the wasteless formula for the pseudodistance from the origin on such domains (Theorem 2.2). We also apply this formula to the case of dimension two and represent the pseudodistance by means of the continued fraction expansion of real numbers (Theorem 4.1).

1. Preliminary.

Let D be a domain in $\boldsymbol{C}^n$. For $p, q \in D$, let

$$c_D^*(p, q)=\sup\{|f(q)|\,;\, f\in \mathrm{Hol}(D, U),\ f(p)=0\},$$

$$k_D^*(p, q)=\inf\{t\,;\, 0\leqq t<1, \text{ there exists an } f\in\mathrm{Hol}(U, D) \text{ such that } f(0)=p \text{ and } f(t)=q\},$$

and

$$g_D(p, q)=\sup\{f(q)\,;\, f \text{ is a negative plurisubharmonic function on } D \text{ such that } \limsup_{z\to p}(f(z)-\log|z-p|)<+\infty\},$$

where U is the unit disc in $\boldsymbol{C}$ and, for complex manifolds X and Y, $\mathrm{Hol}(X, Y)$ denotes the set of all holomorphic mappings from X into Y. The function $c_D=\tanh^{-1}c_D^*$ (resp. the largest pseudodistance k_D on D dominated by $k_D':=\tanh^{-1}k_D^*$) is called the Carathéodory (resp. Kobayashi) pseudodistance on D, and the function $g_D(p, \cdot)$ is called the pluri-complex Green function on D with pole at p (cf., e.g., [2], [3], [4], [5], [10], [11], [15]). These functions c_D^*, k_D^*, and g_D have the decreasing property for holomorphic mappings and satisfy

$$c_D^*\leqq \exp g_D\leqq k_D^* \quad \text{on } D\times D \tag{1.1}$$

(see [10]).

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