K. TOHGE
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ON THE VALUE DISTRIBUTION OF AN ENTIRE FUNCTION OF ORDER AT MOST ONE

BY KAZUYA TOHGE

§1. Introduction.

As a consequence of results on solutions to a differential equation $w''+Aw=0$, where A is entire, Shen [5] and Rossi [4] proved a curious result:

There does not exist a transcendental entire function E of order $\rho(E)<1$ such that the value of $E'(z)$ at every zero of $E(z)$ is ± 1.

On proving this, they used the lemma of Bank and Laine [1] which states that such a function E would have to be the product of two linearly independent solutions of the above second order differential equation. It follows from the counter-example given by Rossi, $E(z)=2\sqrt{z}\sin\sqrt{z}$, that the conclusion can not hold even if only one zero fails to satisfy the assumption.

In this note we prove

THEOREM. *Let $E(z)$ be a transcendental entire function of order $\rho(E)\leqq 1$ and $Q(z)\not\equiv 0$ a rational function. Suppose that $E'(z)-Q(z)$ vanishes at every zero of $E(z)$ with possibly finitely many exceptions. Then $\rho(E)=1$ and further E is of regular growth, and also the meromorphic function*

$$A(z)=\frac{E'(z)-Q(z)}{E(z)} \tag{1.1}$$

is one of the followings:

a) *A is a rational function such that for some nonzero constant a, $A(z)\to a$ as $z\to\infty$;*

b) *A is a transcendental function of regular growth with $\rho(A)=1$, and has a finite number of poles.*

This result may be read as a result on the zeros of $E'(z)$.

We can easily give examples for the case a).

Example 1. For any polynomials $p\not\equiv 0$ and q and also a nonzero constant

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