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ON WEAKLY STABLE YANG-MILLS FIELDS OVER POSITIVELY PINCHED MANIFOLDS AND CERTAIN SYMMETRIC SPACES

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Abstract

In this paper it is proved that for $n \geq 5$ there exists a constant $\delta(n)$ with $1/4 < \delta(n) < 1$ such that any weakly stable Yang-Mills connection over a simple connected compact Riemannian manifold M of dimension n with $\delta(n)$-pinched sectional curvatures is always flat. The pinching constants are possible to compute by elementary functions. Moreover we give some remarks on stability of Yang-Mills connections over certain symmetric spaces.

Introduction.

Let M be an n-dimensional compact Riemannian manifold with a metric g and G be a compact Lie group with the Lie algebra g. Let E be a Riemannian vector bundle over M with structure group G, and let $\mathcal{C}_E$ denote the space of G-connections on E, which is an affine space modeled on the vector space $\Omega^1(g_E)$ of smooth 1-forms with values in the adjoint bundle g_E of E. The Yang-Mills functional $\mathcal{YM}: \mathcal{C}_E \to \boldsymbol{R}$ is

$$\mathcal{YM}(\nabla) = \frac{1}{2}\int_M \|F^{\nabla}\|^2 dvol,$$

for each $\nabla \in \mathcal{C}_E$, where F^{∇} is the curvature form of the connection ∇. Note that F^{∇} is a smooth section of $\Omega^2(g_E)$. The Yang-Mills connection $\nabla \in \mathcal{C}_E$ is a critical point of $\mathcal{YM}$. A Yang-Mills connection ∇ is called *weakly stable* if, for each $\nabla^t \in \mathcal{C}_E$ with $\nabla = \nabla^0$,

$$(d^2/dt^2)\mathcal{YM}(\nabla^t)|_{t=0} \geqq 0 .$$

M is called *Yang-Mills unstable* (cf. [K-O-T]) if, for every vector bundle (E, G) over M, any weakly stable Yang-Mills connection on E is always fiat. First Simons proved that the Euclidean n-sphere S^n for $n \geqq 5$ is Yang-Mills unstable ([B-L]). Ever since several persons have investigated the instability of Yang-Mills fields over various Riemannian manifolds; convex hypersurfaces, submani-

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