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INTERPOLATING SEQUENCES ON PLANE DOMAINS

Dedicated to Professor T. Fuji'i'e on his 60th birthday

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1. Let D be an arbitrary open set in $\boldsymbol{C}$, and $H^\infty(D)$ be the set of all bounded analytic functions on D. A sequence $\{z_j\}$ in D is called $(H^\infty(D)$-) interpolating sequence if for all bounded sequences $\{a_j\}$, the interpolation problem

$$f(z_j)=a_j, \qquad j=1, 2, 3, \cdots$$

has a solution $f(z)$ in $H^\infty(D)$. Since L. Carleson characterized the interpolating sequence in the unit disk in [3], several authors have studied interpolating sequences in more general plane domains (for example [2], [7]). In this paper we study interpolating sequences in an arbitrary plane domain and in a certain plane domain satisfying some geometrical condition.

In §2, we show the following theorem by which we see that the interpolating sequence can be characterized in terms of its local behavior.

THEOREM 1. *Let $S=\{z_j\}$ be a sequence in an arbitrary open set D of the complex plane $\boldsymbol{C}$. If for all $\zeta\in\boldsymbol{C}$, there exists some neighborhood U of ζ such that $S\cap U$ is an $H^\infty(D\cap U)$-interpolating sequence, then S is an $H^\infty(D)$-interpolating sequence.*

When $\{z_j\}$ is an $H^\infty(D)$-interpolating sequence, the open mapping theorem shows that

$$M=\sup_{\|\{a_j\}\|_\infty\leq 1}\inf\{\|f\|_\infty : f\in H^\infty(D),\ f(z_j)=a_j,\ j=1, 2, 3, \cdots\}<\infty,$$

where $\|\{a_j\}\|_\infty=\sup|a_j|$. This constant M is called the constant of interpolation for $\{z_j\}$ (in D). For $\Delta=\{|z|<1\}$, L. Carleson [3] showed that a sequence $\{z_j\}$ in Δ is the interpolating sequence if and only if

$$\delta=\inf_k \prod_{\substack{j=1\\ j\neq k}}^{\infty}\left|\frac{z_k-z_j}{1-\bar{z}_k z_j}\right|>0.$$

The constant of interpolation M for $\{z_j\}$ is estimated by the inequalities (see

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