S. TANNO
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INTEGRALS OF SOME TRIGONOMETRIC FUNCTIONS

BY SHUKICHI TANNO

1. Introduction

Let a and b be positive numbers, and u, v and m be positive integers such that $u+v=m$, $2\leq m$. We define $I(m)$ and $I(u;v)$ by

$$I(m)=\int_0^\infty \frac{\sin^m at}{t^m}dt, \qquad I(u;v)=\int_0^\infty \frac{\sin^u at \sin^v bt}{t^m}dt.$$

Tables of integrals give values of $I(m)$ and $I(u;v)$ only for some special cases. For example, for $3\leq m\leq 6$, Gradshteyn and Ryzhik [4] (p. 449-p. 450) gives

$$I(3)=3a^2\pi/8, \qquad I(4)=a^3\pi/3,$$
$$I(5)=115a^4\pi/384, \qquad I(6)=11a^5\pi/40.$$

As for $I(u;v)$ with $a<b$, [4] (p. 451-p. 452) gives

$$I(2;2)=(3b-a)a^2\pi/6 \qquad (a<b),$$
$$I(3;1)=a^3\pi/2 \qquad (0<3a\leq b)$$
$$=[24a^3-(3a-b)^3]\pi/48 \qquad (0<a<b\leq 3a),$$
$$I(1;3)=(9b^2-a^2)a\pi/24 \qquad (a<b).$$

In this note we give the general expressions of $I(m)$ and $I(u;v)$. These are special cases of Theorem A below. To state our Theorem A we need the following definition. Let $a_1, a_2, \cdots, a_m$ be positive numbers such that $0<a_1\leq a_2\leq \cdots \leq a_{m-1}\leq a_m$. For a subset $\lambda=\{k_1, k_2, \cdots, k_{m-r}\}$ of $\{1, 2, \cdots, m-1\}$, a polynomial

$$P_r(\lambda)=a_{k_1}+a_{k_2}+\cdots+a_{k_{m-r}}-a_{k_{m-r+1}}-\cdots-a_{k_{m-1}}-a_m$$

is said to be of *r-type*, if $\{a_{k_1}, a_{k_2}, \cdots, a_{k_{m-1}}, a_m\}=\{a_1, a_2, \cdots, a_m\}$ as sets and

$$P_r(\lambda)>0, \quad k_1<k_2<\cdots<k_{m-r}, \quad k_{m-r+1}<\cdots<k_{m-1}.$$

Note that a_m appears with negative sign and r is the number of negative signs contained in a polynomial of r-type. A polynomial of 1-type is unique if it exists.

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