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# ON THE ZERO-ONE-POLE SET OF A MEROMORPHIC FUNCTION, II

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**0.** Let $\{a_n\}$, $\{b_n\}$ and $\{p_n\}$ be three disjoint sequences with no finite limit points. If it is possible to construct a meromorphic function $f$ in the plane $C$ whose zeros, $d$-points and poles are exactly $\{a_n\}$, $\{b_n\}$ and $\{p_n\}$ respectively, where their multiplicities are taken into consideration, then the given triad $(\{a_n\}, \{b_n\}, \{p_n\})$ is called a *zero-d-pole set*. Here of course $d$ is a nonzero complex number. Further if there exists only one meromorphic function $f$ whose zero-$d$-pole set is just the given triad, then the triad is called *unique*. It is well known that unicity in this sense does not hold in general.

In Sections 1 and 2, the letter $E$ will denote sets of finite linear measure which will not necessarily be the same at each occurrence.

**1.** Let $f$ and $g$ be meromorphic functions in the plane $C$. If $f$ and $g$ assume the value $a\in C\cup\{\infty\}$ at the same points with the same multiplicities, we denote this by $f=a\Leftrightarrow g=a$. With this notation, our first result of this note is stated as follows.

THEOREM 1. *Let $f$ and $g$ be nonconstant meromorphic functions satisfying $f=0\Leftrightarrow g=0$, $f=1\Leftrightarrow g=1$ and $f=\infty\Leftrightarrow g=\infty$. If*

$$(1.1)\qquad \bar{K}(f)=\limsup_{r\to\infty}\{\bar{N}(r, 0, f)+\bar{N}(r, \infty, f)\}/T(r, f)<1/2,$$

*then $f\equiv g$ or $fg\equiv 1$.*

From this we immediately deduce the following

COROLLARY 1. *Let $f$ be a nonconstant meromorphic function satisfying $n(r, 0, f)+n(r, \infty, f)\not\equiv 0$ and $\bar{K}(f)<1/2$. Then the zero-one-pole set of $f$ is unique.*

The estimate (1.1) is sharp. For example, let us consider $f=e^{\alpha}(1-e^{\alpha})$ and $g=e^{-\alpha}(1-e^{-\alpha})$ with a nonconstant entire function $\alpha$. Then we easily see that $f=0\Leftrightarrow g=0$, $f=1\Leftrightarrow g=1$ and $f=\infty\Leftrightarrow g=\infty$. Also, $f\not\equiv g$ and $fg\not\equiv 1$ are evident.

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