SOME PROPERTIES OF THE ENTIRE FUNCTIONS
EXTREMAL FOR DENJOY'S CONJECTURE

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1. Introduction

In this paper we shall prove the following

THEOREM 1. *Let $F(Z)$ be an entire function extremal for Denjoy's Conjecture (that is, F is entire of finite order λ and has $k=2\lambda$ distinct finite asymptotic values) and satisfy the condition $\varlimsup_{r\to\infty} \log M(r, F)/r^{k/2} < \infty$, then $F(Z)$ is right-prime.*

THEOREM 2. *Let $F(Z)$ be an entire function extremal for Denjoy's Conjecture and $P(Z)$ a nonconstant polynomial whose zeros are distinct from zeros of $F(Z)$, then $F(Z)/P(Z)$ is right-prime.*

THEOREM 3. *Let $A(Z)$ be an entire function extremal for Denjoy's Conjecture and f_1, f_2 two linear independent solutions of $f''+Af=0$, then at least one of f_1, f_2 has the property that the exponent of convergence of its zero-sequence is ∞.*

In 1907, A. Denjoy [1] posed the following famous conjecture:
Let $F(Z)$ be an entire function of finite order λ, if it has K distinct finite asymptotic values, then $K \leq 2\lambda$.
L. Ahlfors [2] confirmed the conjecture in 1930.
An entire function $F(Z)$ is called to be extremal for Denjoy's conjecture $K \leq 2\lambda$ if it is of finite order λ and has $K=2\lambda$ distinct finite asymptotic values. Since then, this kind of functions extremal for Denjoy's Conjecture was investigated by many mathematicians such as L. Ahlfors [2] P. Kennedy [3] D. Drasin [4] and Guang-hou Zhang [5]. Here we consider some other properties of this kind of functions.

2. Preliminary and lemmas

First, we introduce the notion of right-prime.
Let F be a meromorphic function on $|Z| < \infty$, if $F(Z)$ can be written as

$$F(Z) = f(g(Z)) \tag{1}$$