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ON THE BOUNDARY BEHAVIOR OF HOLOMORPHIC MAPPINGS OF PLANE DOMAINS INTO RIEMANN SURFACES

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1. Since the pioneering work of Ohtsuka ([7], [8]), several papers have dealt with Picard-type theorems for holomorphic mappings of plane domains into Riemann surfaces. See, *e.g.*, [2], [4], [5], [6], [9], [10], [12] and [13]. In this note we shall consider the behavior around null-sets of class N_B and N_D (in the familiar notation of Ahlfors-Beurling [1]) of holomorphic mappings into certain Riemann surfaces. Our result concerning the class N_B (Theorem 1) can be regarded as a generalization of a recent result of Shiga [12, Theorem 2].

2. We begin with some terminology. Let W be a Riemann surface and $E\subset W$ a compact totally disconnected set. We say that E is of class N_B (resp. N_D) in W if for each $p\in E$ there is a parametric disc (V, φ) in W such that $p\in V$, $E\cap\partial V=\emptyset$ and $\varphi(E\cap V)$ is of class N_B (resp. N_D). Let W^* stand for the Stoïlow compactification of W, and let $p\in\beta=W^*\setminus W$. We say that p is AB-removable (resp. AD-removable) if there is a planar end $V\subset W$ with $p\in\beta_V$, the relative ideal boundary of V, and a conformal map φ of $\bar{V}$ into the closed unit disc $\bar{U}\subset \boldsymbol{C}$ such that $\varphi(\partial V)=\partial\bar{U}$ and $\bar{U}\setminus\varphi(\bar{V})$ is of class N_B (resp. N_D). Obviously, $p\in\beta$ is AB-removable if and only if there is a Riemann surface $W'\supset W$ such that $p\in W'\setminus W$ and $W'\setminus W$ is of class N_B in W'. As usual, $\mathcal{O}_{AB}$ denotes the class of Riemann surfaces which do not carry nonconstant bounded holomorphic functions, while $\mathcal{O}_{MD^*}$ stands for the class of Riemann surfaces without non-constant meromorphic functions with a finite spherical Dirichlet integral.

THEOREM 1. *Let D be a plane domain and let $E\subset D$ be a compact set of class N_B. Let W be a Riemann surface which does not belong to $\mathcal{O}_{AB}$, and let $f: D\setminus E\to W$ be a holomorphic mapping. Then there exists a Riemann surface $W'\supset W$ such that*

(a) *$W'\setminus W$ is of class N_B in W' and*
(b) *f extends to a holomorphic mapping $f^*: D\to W'$.*

Proof. We may assume that f is nonconstant. Let g be a nonconstant bounded holomorphic function in W. Since $E\in N_B$, $g\circ f$ admits a holomorphic

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