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CERTAIN PROPERTIES OF $S(x, n)$

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Let $S(x, n)$ be the function of x and n defined as follows:

$$(1)\qquad S(x, n)=-BS_3(x, n)+(n-x)^{n-1}S_4(x, n),$$

where $B=(n-1)^{n-1}$,

$$(2)\qquad S_3(x, n)=(8n^2-5)x^3-2(8n^3+20n^2-15n+20)x^2 +3(24n^3-68n^2+42n-5)x+4n(4n-1)(4n-3)$$

and

$$(3)\qquad S_4(x, n)=(n-1)(4n^2-10n+5)x^4+(8n^3-52n^2+87n-40)x^3 +3(12n^3-42n^2+37n-5)x^2+3n(16n^2-32n+9)x+12n^2(2n-1).$$

The present author proved the following facts:

FACT 1. *$S(x, n)>0$ for $0\leqq x\leqq n$, $x\neq 1$, with $n\geqq 2$*

(Proposition 4 in [1]);

FACT 2. *$S(x, n)$ is decreasing in $0<x<1$ with $n\geqq 2$, and increasing in $1<x<n$ with $2\leqq n\leqq \dfrac{11+\sqrt{77}}{4}=4.9437410\cdots$*

(Proposition 8 in [2]).

The proof of the second part of Fact 2 was too long and worked out elaborately even though it was expected with $n\geqq 2$. We shall give another proof of it with $n\geqq 2$.

MAIN THEOREM. *$S(x, n)$ is increasing in $1<x<n$ with $n\geqq 2$.*

By means of the argument of §3 in [2], setting $x=1+y$ and $n=1+m$, we have from (1) and (2)

$$S_3(1+y)=(8m^2+16m+3)y^3-(16m^3+64m^2-22m-15)y^2 +10m(4m^2-14m-7)y+60m^2(2m+1),$$

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