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# ENTIRE FUNCTIONS WITH RADIALLY DISTRIBUTED ZEROS

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**1.** In our previous paper [1], we considered the entire functions of positive integral order and obtained the following characterization of the exponential function.

THEOREM A. *Suppose that $f(z)$ is an entire function of positive integral order $p$, and that $f(z)$ has no zeros in a sector $\{z;|\arg z|<\pi-\pi/2p+\eta\}$ $(\eta>0)$ and $\delta(0, f)=1$. If there exists a Jordan curve $l$ joining $z=0$ to $z=\infty$ such that*

$$f(z)f(\omega z)\cdots f(\omega^{2p-1}z)=O(1) \qquad (z\in l)$$

*where $\omega=\exp(\pi i/p)$, then $f(z)=e^{P(z)}$ where $P(z)$ is a polynomial of degree $p$, or else*

$$\lim_{r\to\infty}\frac{|\log|f(r)||}{r^{p}}=+\infty .$$

In this paper, we show that we can remove the condition on the deficiency. But we confine the distribution of zeros in a sector with half opening and prove the following.

THEOREM 1. *Suppose that $f(z)$ is an entire function of positive integral order $p$, and that $f(z)$ has only zeros in a sector $\{z;|\arg z-\pi|\leqq\pi/4p-\eta=\alpha\}$ $(\eta>0)$. If there exists a Jordan curve $l$ joining $z=0$ to $z=\infty$ such that*

$$f(z)f(\omega z)\cdots f(\omega^{2p-1}z)=O(1) \qquad (z\in l) \tag{1}$$

*where $\omega=\exp(\pi i/p)$, then $f(z)=e^{P(z)}$ where $P(z)$ is a polynomial of degree $p$, or else*

$$\lim_{r\to\infty}\frac{|\log|f(r)||}{r^{p}}=+\infty . \tag{2}$$

In our previous paper [1], we also considered the entire function of order $q=2p+1$ having only negative zeros and obtained the following characterization of the exponential function.

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