W. BERGWEILER
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ON MEROMORPHIC FUNCTIONS THAT SHARE THREE VALUES AND ON THE EXCEPTIONAL SET IN WIMAN-VALIRON THEORY

BY WALTER BERGWEILER[1)]

1. Introduction and Results

Two meromorphic functions are said to share a value a if they have the same a-points. We distinguish the cases that we count multiplicities (CM) and that we ignore multiplicities (IM). One of the main tools that has been used in the study of functions that share values is Nevanlinna's theory on the distribution of values (cf. [3, 6, 7]). Here it is important to have relations between the Nevanlinna characteristics $T(r, f)$ and $T(r, g)$ if f and g share values.

It is well-known [2, Theorem 2; 5, Satz 3] that

$$T(r, f)\sim T(r, g) \qquad (r\notin E) \tag{1.1}$$

if f and g share four values IM. Here and in the following E denotes an exceptional set of finite measure. If f and g share three values IM, then

$$\frac{1}{3}-o(1)\leqq\frac{T(r, f)}{T(r, g)}\leqq 3+o(1) \qquad (r\notin E). \tag{1.2}$$

This was proved by Gundersen [2, Theorem 3] who also gave an example which shows that the bounds 1/3 and 3 are sharp.

This paper is concerned with the question what can be said about the relation between $T(r, f)$ and $T(r, g)$ if f and g share three values CM. A recent result of Brosch [1, Satz 5.7] says that (1.2) can be improved in this case. He proved that

$$\frac{3}{8}-o(1)\leqq\frac{T(r, f)}{T(r, g)}\leqq\frac{8}{3}+o(1) \qquad (r\notin E). \tag{1.3}$$

It is not known whether these bounds are sharp. Osgood and Yang [8, Theorem 3] proved that $T(r, f)\sim T(r, g)$ if f and g are entire functions of finite order and they conjectured that this remains true for arbitrary entire functions. The

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