P. COULTON AND H. GAUCHMAN
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SUBMANIFOLDS OF QUATERNION PROJECTIVE SPACE WITH BOUNDED SECOND FUNDAMENTAL FORM

BY PARTICK COULTON AND HILLEL GAUCHMAN

Abstract. Let h be the second fundamental form of a compact submanifold M of the quaternion projective space $\boldsymbol{HP}^n(1)$. For any unit vector $u\in TM$, set $\delta(u)=\|h(u, u)\|^2$. We determine all compact totally complex submanifolds of $\boldsymbol{HP}^n(1)$ (resp. all compact totally real minimal submanifolds of $\boldsymbol{HP}^n(1)$) satisfying condition $\delta(u)\leqq\frac{1}{4}$ (resp. $\delta(u)\leqq\frac{1}{12}$) for all unit vectors $u\in TM$.

1. Introduction.

Let M be a smooth m-dimensional Riemannian manifold isometrically immersed in an $(m+p)$-dimensional Riemannian manifold $\tilde{M}$. Let h denote the second fundamental form of this immersion. For each $x\in M$, h is a bilinear mapping from $TM_x\times TM_x$ into $TM_x^{\perp}$, where TM_x is the tangent space of M at x and $TM_x^{\perp}$ is the normal space. We denote by $S(x)$ the square of the length of h at $x\in M$. By Gauss' equation we have $S(x)=m(m-1)-\rho(x)$, whenever M is immersed as a minimal submanifold of $S^{m+p}(1)$ with scalar curvature $\rho(x)$ at x in M. Therefore $S(x)$ is an intrinsic invariant of M.

In 1968, J. Simons [12] discovered for the class of compact minimal m-dimensional submanifolds of the unit $(m+p)$-sphere that the totally geodesic submanifolds are isolated in the following sense: If $S(x)<n/(2-1/p)$ for all $x\in M$, then $S(x)\equiv 0$ on M, and thus M is totally geodesic. In [1], S. S. Chern, M. do Carmo, and S. Kobayashi determined all minimal submanifolds of the unit sphere satisfying $S(x)\equiv n/(2-1/p)$. Later similar results were obtained for various types of minimal submanifolds of the complex projective spaces and the quaternion projective spaces.

Let $T: UM\rightarrow M$ and UM_x denote the unit tangent bundle of M along with its fibre over $x\in M$. We set $\delta(u)=\|h(u, u)\|^2$ for $u\in UM$. Observe that $\delta(u)$ is not an intrinsic invariant of the submanifold M. However, like $S(x)$, $\delta(u)$ can be considered as a natural measure of the degree to which an immersion fails to be totally geodesic.

In [10], and [11], A. Ros proved that if M is a compact Kaehler submanifold of $\boldsymbol{CP}^n(1)$ and if $\delta(u)<1/4$, for any $u\in UM$, then M is totally geodesic in

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