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SELF-HOMOTOPY EQUIVALENCES OF $H_*(-; Z/p)$-LOCAL SPACES

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Abstract

Under certain finiteness conditions, p-completion commutes with the formation of a certain group of self-homotopy equivalences.

1. Introduction.

Let X be a pointed, 0-connected topological space, $\mathrm{Aut}(X)$ the group of based homotopy classes of based self-homotopy equivalences of X, and $\mathrm{Aut}_\#(X)$ the kernel of the obvious homomorphism

$$\mathrm{Aut}(X) \longrightarrow \prod_{i=1}^{d} \mathrm{Aut}\, \pi_i(X)$$

where we further assume either that X is a CW-complex of dimension d or that $\pi_*(X)=0$ for $*>d$, $1 \leqq d < \infty$. The purpose of this paper is to investigate the behaviour of $\mathrm{Aut}_\#$ under $H_*(-; \boldsymbol{Z}/p)$-localization of the space.

To explain the main result, let X be a finite, connected, and nilpotent CW-complex and $X_{Z/p}$ its $H_*(-; \boldsymbol{Z}/p)$-localization in the sense of Bousfield [1]. Then $\mathrm{Aut}_\#(X)$ is nilpotent [3] and

$$\mathrm{Aut}_\#(X_{Z/p}) = \mathrm{Ext}(\boldsymbol{Z}/p^\infty, \mathrm{Aut}_\#(X))$$

where $\mathrm{Ext}(\boldsymbol{Z}/p^\infty, -)$ is the Ext$-p$-completion functor defined for all nilpotent groups by Bousfield and Kan [2].

This paper can be viewed as a parallel, not only in subject but also in method, to [7]. I am very grateful to Prof. K. Maruyama for sending me a preprint of his paper, to Prof. C. U. Jensen who kindly supplied the proof of Proposition 2.4, and to the topologists at Memorial University for the invitation to their conference on Spaces of Self-homotopy Equivalences, Montreal, August 1988.

2. Completions of nilpotent groups.

In this section I collect for later reference some fundamental facts about

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