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DUPIN HYPERSURFACES AND A LIE INVARIANT

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Dedicated to Professor Tominosuke Otsuki on his 70th birthday

§ 1. Introduction.

In [9], Pinkall developes a Lie's sphere geometry on hypersurfaces in a space form and successfully applies the theory to a class of hypersurfaces called "Dupin". A Dupin hypersurface is a hypersurface each of which principal curvatures has a constant multiplicity with a vanishing derivative in the corresponding curvature direction. One of his results is the local Lie equivalence of cyclides of Dupin with isoparametric hypersurfaces, where a cyclide of Dupin is a Dupin hypersurface with exactly two principal curvatures. This is essentially used in [4] to find a solution to a simple progressing wave equation.

For any integer g, we can construct a Dupin hypersurface with g principal curvatures of arbitrary multiplicities. Isoparametric hypersurfaces, however, have $g \in \{1, 2, 3, 4, 6\}$ principal curvatures with non-arbitrary multiplicities if $g \geqq 3$. Thus the equivalence problem between Dupin hypersurfaces and isoparametric hypersurfaces for $g \geqq 3$ requires some more conditions on Dupin hypersurfaces.

In [11], Thorbergsson guarantees coincidence of *compact embedded* Dupin hypersurfaces with isoparametric hypersurfaces in cohomology level. This motivates Cecil and Ryan a conjecture [3]: A compact embedded Dupin hypersurface is Lie equivalent to an isoparametric hypersurface. Besides the trivial case $g=1$, this is already known true when $g=2$ [2]. For $g=3$, the author gives a positive answer in [4]. In this paper, we find a certain Lie invariant by which we get a non-trivial necessary condition for the equivalence when $g=4$ and 6. A sufficient condition for $g=4$ is obtained as well, and in the forthcoming paper, we give it for $g=6$.

After this paper was finished, Pinkall and Thorbergsson construct counterexamples to the conjecture for $g=4$ [10]. Independently, Ozawa and the author get counterexamples for $g=4$ and 6 in [6], using a new method producing taut embeddings. Both examples are shown to be not Lie equivalent to isoparametric hypersurfaces by using the Lie invariant obtained in the present paper.

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