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PERIODIC EXTENSIONS OF TWO-DIMENSIONAL BROWNIAN MOTION ON THE HALF PLANE, I

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Introduction

In the paper and the following one [6], we shall study periodic extensions of the Brownian motion on the half plane $\bar{D}=\{(x, y): y\geqq 0\}$. By an "extension", we mean a Markov process on $\bar{D}$ whose laws before paths reach the boundary $\partial_0=\{(x, y): y=0\}$ coincide with those of the two-dimensional Brownian motion, and by a "periodic extension" we mean an extension whose laws are invariant under translation of length 2π parallel to the x-axis.

First, let us quote an extension as an example. Assume the semigroup of the extension satisfies the boundary condition

$$\alpha(x)\frac{\partial^2}{\partial x^2}+\beta(x)\frac{\partial}{\partial x}+\frac{\partial}{\partial y}=0$$

on ∂_0, where α and β are smooth periodic functions on the real line and α is positive. Then, the extension is periodic, has continuous paths and has no sojourn on the boundary ∂_0. Let functions u and m be harmonic in $D=\{(x, y): y>0\}$ (in classical sense) and smooth in $\bar{D}$, and satisfy

$$\alpha(x)u_{xx}(x, 0)+\beta(x)u_x(x, 0)+u_y(x, 0)=0$$

$$u(x+2\pi, y)-u(x, y)=2\pi$$

$$(\alpha(x)m(x, 0))_{xx}-(\beta(x)m(x, 0))_x+m_y(x, 0)=0$$

and

$$\int_0^{2\pi} m(x, y)dx=2\pi\,.$$

Such smooth functions u and m with $u_x>0$ and $m>0$ uniquely exist. Define

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