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INTERPOLATION THEOREM IN MANY-VALUED LOGICS WITH DESIGNATED VALUES

Dedicated to Professor S. Maehara on his 60th birthday

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§ 0. Introduction.

Many-valued logics can be divided into two categories according as the designated values are specified or not. In our research on logics in which the designated values are specified, we are concerned ourselves with the validity of formulas if anything. On the other hand, we are obliged to deal with the validity of sequents introduced by Rousseau [6], Takahashi [7] and originally by Gentzen, in the study of those in which the designated values are not specified. Consequently, many-valued versions of Craig's interpolation theorem are also separated into two groups. Gill's version for a 3-valued logic in [1] and Rasiowa's for an m-valued logic ($m>2$) in [5], not forgetting Craig's original theorem for the classical logic belong to the first group. In the meantime, examples of those of the second group are Miyama's version (see Fact 1 of this paper) in [4] and that (see Fact 2) by Hanazawa together with the present author in [2]; both followed Maehara who in [3] had given an elegant proof of Craig's theorem utilizing the cut-elimination theorem of Gentzen (see Takeuti [8]). As a by-product of syntactical proof of Rasiowa's, Wasilewska also obtained a version of the second group in [9].

In this paper, a (finitely) many-valued version of Craig's interpolation theorem of the first group will be investigated in a rather general setting. We remark that the choice of (the truth-value function of) "implication" is most problematic.

Suppose that a many-valued logic L is given. Let $\boldsymbol{T}$ and $\boldsymbol{D}$ be the sets of truth-values and of designated values of L, respectively. We assume that L is functionally complete in a strong sense, that is, for every function $f: \boldsymbol{T}^k \to \boldsymbol{T}$ there is a formula $f^*(P_1, \cdots, P_k)$ representing f for each nonnegative integer k, and for every function $q: \{X \mid \emptyset \neq X \subseteqq \boldsymbol{T}\} \to \boldsymbol{T}$ there is a formula $q^* x Q(x)$ representing q. Now we propose the following problem: *Find the necessary and sufficient condition on* $f: \boldsymbol{T}^2 \to \boldsymbol{T}$ *for which the interpolation property with respect to* f *formulated below holds.*

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