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THE SINGULAR DIRICHLET PROBLEM FOR THE COMPLEX MONGE-AMPÈRE OPERATOR ON COMPLEX MANIFOLDS

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1. Introduction.

Let M be a connected paracompact complex manifold of dimension n with a fixed volume form dV, and Ω a relatively compact, strictly pseudoconvex open subset of M. Bedford and Taylor [3] showed that

$$(dd^c u)^n := \wedge^n dd^c u$$

is well defined as a positive Radon measure for a locally bounded, plurisubharmonic function u on Ω, where $d^c = \sqrt{-1}\,(\bar\partial - \partial)$. We call the assignment $u \to (dd^c u)^n$ the complex Monge-Ampère operator. In this paper we study the nonlinear $(n>1)$ Dirichlet problem for the complex Monge-Ampère operator:

$$\begin{aligned} &u \text{ is plurisubharmonic on } \Omega, \\ &\lim_{z\to\partial\Omega} u(z) = \phi \qquad \text{on } \partial\Omega, \\ &(dd^c u)^n = F(u, z)\,dV \qquad \text{on } \Omega, \end{aligned} \tag{1.1}$$

where ϕ is a real-valued function on $\partial\Omega$ and F a non-negative function on Ω. Many results on the existence and the regularity of the solution of (1.1) were obtained in [1], [3], [4], [5], [7], [8], [9], [11], [12], [13], etc. In the case of singular boundary data (i.e., $\phi = +\infty$), however, the singular Dirichlet problem seems to be unknown except for some special cases (for example, [5], [10], [14]), some of which are treated in the context of the existence of the complete Kähler-Einstein metric. We will show the existence of generalized solution of the equation (1.1) for the singular boundary data on Stein manifold, and extend Theorem 5 in Bedford and Taylor [5], which states that if Ω is a bounded strictly pseudoconvex set in $\boldsymbol{C}^2$, $F \in C(\boldsymbol{R} \times \bar\Omega)$, $F \geqq 0$, $t \to F(t, z)$ increasing in t, $t \to [F(t, z)]^{1/2}$ a convex function of t, and F has an upper barrier, then $u(z) := \sup\{v(z) : v \in P(\Omega) \cap L^\infty_{\mathrm{loc}}(\Omega),\ \Phi(v) \geqq [F(t, z)]^{1/2}\}$ is a solution of (1.1) with $\phi = +\infty$. As a result we will establish

THEOREM. *Let M be an n-dimensional Stein manifold with volume form dV and Ω a relatively compact, C^0 strictly pseudoconvex domain in M. Let $F \in L^\infty_{\mathrm{loc}}$*

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