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SUBSPACES OF TRIGONAL RIEMANN SURFACES

Dedicated to Professor Kôtaro Oikawa on his sixtieth birthday

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1. Introduction.

In the present paper, we shall study on relations between subspaces of the space of trigonal Riemann surfaces of genus $g \geqq 5$.

Recently, the author and Horiuchi [6] have studied on the Weierstrass gap sequences at the ramification points of trigonal Riemann surfaces. It was also studied by Coppens [1, 2, 3]. Coppens' study in [2] depends upon the fact that any trigonal Riemann surface lies on a rational normal scroll. On the other hand, the author and Horiuchi's study depends upon the fact that any trigonal Riemann surface is defined by an algebraic equation in x and y whose degree is three with respect to y. They determined a canonical equation of a trigonal Riemann surface of genus g and of the n-th kind and gave the necessary and sufficient condition for determining the types of ramification points in terms of zeros of the discriminant of the defining equation.

At first, we shall give an algebraic equation:

$$y^3+Q(x)y+R(x)=0.$$

Let S be the trigonal Riemann surface defined by the equation. We shall decide the genus and the kind of S and the types of the ramification points.

Using this result, we obtain incidence relations between $M_{g,3,n}(\rho_1, \rho_2, \rho_3, \rho_4)$'s. The definition of $M_{g,3,n}(\rho_1, \rho_2, \rho_3, \rho_4)$ will be given later.

Let S be a trigonal Riemann surface of genus g and let $x: S \to \mathbf{P}^1$ be a trigonal covering. Following Coppens [1] we say that S is of the n-th kind if $l(nD)=n+1$ and $l((n+1)D) \geqq n+3$, where $D=(x)_\infty$ is the polar divisor of x, $l(nD)$ (resp. $l((n+1)D)$ is the affine dimension of the space of meromorphic functions on S whose divisors are multiples of nD (resp. $(n+1)D$) and n satisfies $(g-1)/3 \leqq n \leqq g/2$.

By definition, a point P on S is a total (resp. an ordinary) ramification point if the ramification index of x at P is equal to three (resp. two). We say that P is a total ramification point of type I (resp. type II) if the gap sequence at P is equal to

$$(1, 2, 4, 5, \cdots, 3n-2, 3n-1, 3n+1, 3n+4, \cdots, 3(g-n-1)+1),$$

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