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HIRZEBRUCH L-HOMOLOGY CLASSES AND THE INTERSECTION FORMULA

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1. Introduction. In [7], Goresky and MacPherson introduced the signature for compact oriented PL-pseudo-manifolds which can be stratified with only strata of even codimension, using the intersection homology theory. Furthermore they defined the Hirzebruch L-homology class. Our main purpose is to prove the intersection formula for Hirzebruch L-homology classes, which is the analogy of the Stiefel-Whitney homology classes' version [11]. By simple calculation, the case of manifolds can be reduced to the product formula for cohomology characteristic classes of bundles.

Let X and Y be compact oriented PL-pseudo-manifolds, possibly with boundary, which can be stratified with only strata of even codimension (cf. [7; §5]). If X and Y are properly PL-embedded in an oriented PL-manifold M, and if they are mutually transverse in M, then the intersection $X\cap Y$ is an orientable PL-pseudo-manifold which can be stratified with only strata of even codimension (cf. Proposition 2.3). Then we denote by $X\cdot Y$ the intersection $X\cap Y$ with the canonical orientation. Let a and b be in $H_*(M, \partial M; Q)$. To state our main theorem, we define $a\cdot b$ by $a\cdot b=[M]\cap(([M]\cap)^{-1}a\cup([M]\cap)^{-1}b)$. Let $f: X\to M$, $g: Y\to M$ and $h: X\cdot Y\to M$ be the inclusions. Our main theorem is the following:

THEOREM. *With the above, the following holds:*

$$f_*L_*(X)\cdot g_*L_*(Y)=h_*L_*(X\cdot Y)\cap l^*(M),$$

where $l^*(M)$ is the L-cohomology class of M.

We recall the definition of the Hirzebruch L-homology classes due to Goresky and MacPherson [7]. Let Ω_*^{ev} be the oriented cobordism ring of compact oriented PL-pseudo-manifolds which can be stratified with only strata of even codimension (cf. [7; §5]). Let X be a compact n-dimensional oriented PL-pseudo-manifold without boundary which can be stratified with only strata of even codimension. Denote by $\sigma(X)$ the signature of X ([7]). Then $\sigma: \Omega_*^{ev}\to Z$ is a ring homomorphism ([8]). We denote by $[X, S^k]$ the set of homotopy classes of maps from X to the k-sphere S^k. Define a map $\theta: [X, S^k]\to \boldsymbol{Z}$ by

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