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ON THE THEOREM OF TUMURA-CLUNIE

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1. Introduction and Main Results.

Let f be a nonconstant meromorphic function in the complex plane. It is assumed that the reader is familiar with the notations of Nevanlinna theory (see, for example [3]). We denote by $S(r, f)$ any function satisfying $S(r, f)= o(T(r, f))$ as $r\to+\infty$, possibly outside a set E of finite linear measure. Throughout this paper we denote by $a_j(z)$ meromorphic functions which satisfying $T(r, a_j)=S(r, f)$ $(j=0, 1, \cdots, n)$. If $a_n\not\equiv 0$, we call

$$P[f]=a_nf^n+a_{n-1}f^{n-1}+\cdots+a_1f+a_0$$

a polynomial in f with degree n. If $n_0, n_1, \cdots, n_k$ are nonnegative integers, we call

$$M[f]=f^{n_0}(f')^{n_1}\cdots(f^{(k)})^{n_k} \tag{1}$$

a differential monomial in f of degree $\gamma_M=n_0+n_1+\cdots+n_k$ and of weight $\Gamma_M =n_0+2n_1+\cdots+(k+1)n_k$. If $M_1, \cdots, M_n$ are differential monomials in f, we call

$$Q[f]=\sum_{j=1}^{n}a_j(z)M_j[f] \tag{2}$$

a differential polynomial in f, and define the degree γ_Q and the weight Γ_Q by $\gamma_Q=\max_{j=1}^{n}\gamma_{M_j}$ and $\Gamma_Q=\max_{j=1}^{n}\Gamma_{M_j}$ respectively. If Q is a differential polynomial, then Q' denotes the differential polynomial which satisfies $Q'[f(z)]=\frac{d}{dz}Q[f(z)]$ for any meromorphic function f. (See, for example, Mues and Steinmetz [4, P 115]).

The following theorem was first stated by Tumura [6] and proved completely by Clunie [1]:

THEOREM A. *Let f and g be entire functions, and*

$$F=a_nf^n+a_{n-1}f^{n-1}+\cdots+a_1f+a_0 \qquad (a_n\not\equiv 0). \tag{3}$$

If $F=be^g$, where $b(z)$ is a meromorphic function satisfying $T(r, b)=S(r, f)$, then

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