J. A. BUJALANCE
KODAI MATH. J.
12 (1989), 1—8

HYPERELLIPTIC COMPACT NON-ORIENTABLE KLEIN SURFACES WITHOUT BOUNDARY

BY J. A. BUJALANCE*

By a compact non-orientable Klein surface (KS) X, we shall mean a compact non-orientable surface together with a dianalytic structure on X [3].

A dianalytic homeomorphism of X onto itself will be called an automorphism. We say that a compact non-orientable KS without boundary X is q-hyperelliptic if and only is there exists an involution Φ of X such that $X/\langle\Phi\rangle$ has algebraic genus q (if $q=0$, X is hyperelliptic, if $q=1$, then X is elliptic-hyperelliptic).

In this paper we characterize the q-hyperellipticity of compact non-orientable KS without boundary by means of non-Euclidean crystallographic groups (NEC groups). Similar characterizations for compact orientable KS without boundary (i.e. Riemann surfaces) or for compact KS with boundary have been obtained in [11], [6], [7], [8]. As a consequence from these results, it is obtained: The bound 84 $(p-1)$ for a group of automorphisms of a non orientable KS without boundary can be reduced to 12 $(p-1)$ for most of q-hyperelliptic KS.

1. Preliminary

In this paper we characterize KS by means of NEC groups. Such a group is a discrete subgroup of the group G of all isometries of the hyperbolic plane, including orientation-reversing ones, with compact quotient space (see [10], [14]).

Each NEC group has a signature, that is

$$(g\,;\pm\,;[m_1,\cdots,m_t]\,;\{(n_{i1}\cdots n_{is_i})i=1\cdots k\}). \qquad (1)$$

The numbers m_i are the proper periods, the brachets $(n_{i1}\cdots n_{is_i})$ are the period-cycles and the n_{ij} are the periods of the period-cycles. This signature determines a presentation of the group. Generators:

i) x_i $i=1,\cdots,r$
ii) e_i $i=1,\cdots,k$
iii) c_{ij} $i=1,\cdots,k\,;\ j=0,\cdots,s_i$
iv) (if sign '+') a_i, b_i $i=1,\cdots,g$
(if sign '−') d_i $i=1,\cdots,g$

* Partially supported by CAICYT

Received April 4, 1988