K. ATSUYAMA
KODAI MATH. J.
11 (1988), 403—424

THE CONNECTION BETWEEN THE SYMMETRIC SPACE $E_8/Ss(16)$ AND PROJECTIVE PLANES

BY KENJI ATSUYAMA

§ 0. Introduction.

Simple Lie groups are already classified, and they have four kinds of infinite series of classical types and have five exceptional types. H. Freudenthal wrote many papers to obtain the geometrical and intuitive image of the exceptional Lie groups (cf. [5]). We have now the same aim as his. Our methods to solve the problem were first devised by B. A. Rozenfeld [7], but he didn't succeed completely in explaining the all cases which contain the exceptional Lie groups. For lack of the associativity in Cayley algebras, his explanations were incomplete (cf. [5]). To justify his assertions, we gave first a unified construction of real simple Lie algebras which were easy to handle directly [1]. Namely we made representative spaces for the exceptional Lie groups. Three symmetric spaces with the types EIII, EVI and EVIII in the E. Cartan's sense were next constructed explicitly as orbits of some projections in the sets of endomorphisms of the Lie algebras. We asked whether several similar properties to projective planes hold in the symmetric spaces by regarding the antipodal sets as lines [2], [3]. In this paper we continue to study the type $E_8/Ss(16)$, where $Ss(16)=\mathrm{Spin}(16)/\boldsymbol{Z}_2$, and we assert that this space is also a projective plane in the wider sense of Theorem 4.16.

§ 1. A construction of real simple Lie algebras.

The coefficient field is the field $\boldsymbol{R}$ of real numbers. Composition algebras are classified and have the seven following types:

	real	complex	quaternion	Cayley
division	$\boldsymbol{R}$	$\boldsymbol{C}$	$\boldsymbol{Q}$	$\mathfrak{C}$
split		$\boldsymbol{C}_s$	$\boldsymbol{Q}_s$	$\mathfrak{C}_s$

Let M^n be the $n\times n$ matrix algebra with coefficients in $\boldsymbol{R}$. Set $\mathrm{tr}(X)=(x_{11}+\cdots+x_{nn})/n$ for $X=(x_{ij})\in M^n$ and let $T: X\to X^T$ be the transposed operator. E is the unit matrix of M^n. If $\mathfrak{A}$ is a composition algebra, it has the

Received March 9, 1988