J.M. MØLLER
KODAI MATH. J.
11 (1988), 372—378

SELF-MAPS ON TWISTED EILENBERG-MACLANE SPACES

BY JESPER MICHAEL MØLLER

1. Introduction.

To any (based) space X is associated the monoid $(\sigma(X, *))$ $\sigma(X)$ of (based) homotopy classes of (based) self-maps of X. This monoid contains as its group of units the group $(\varepsilon(X, *))$ $\varepsilon(X)$ of (based) homotopy classes of (based) homotopy equivalences of X.

Let π be any group, A a $\boldsymbol{Z}(\pi)$-module, and denote by $L:=L(A, n)$, $n\geqq 2$, the unique homotopy type with $\pi_1(L)=\pi$, $\pi_n(L)=A$, $\pi_i(L)=0$ for $i\neq 1, n$, that realizes A as a $\pi_1(L)=\pi$-module and has k-invariant $k=0\in H^{n+1}(\pi, A)$.

The purpose of this note is to determine $\sigma(L, *)$ and $\sigma(L)$ explicitly in terms of group theoretic invariants, see Theorems 3.2 and 3.4.

The monoid $\sigma(L, *)$, or rather its subgroup of units $\varepsilon(L, *)$, has attracted some interest in recent years [7], [9], [10], [1], [2] but as far as I know, no explicit formula has been given, at least not in the case of a non-abelian fundamental group.

Throughout this note, I use the notation of [8]: If (X, A) is a pair of spaces, $p: Y\to B$ a fibration, and $u: X\to Y$ a continuous map, then $F_u(X, A; Y, B)$ is the space, equipped with the compactly generated topology associated to the compact-open topology, of all maps $v: X\to Y$ such that $v|A=u|A$ and $pv=pu$. An empty space in the A-entry or a one-point space in the B-entry will be omitted; thus e.g. $F_1(X, *; X)$ is the space of all based self-maps of X.

2. Strategy of proof.

Let $\omega: E\pi\to B\pi$ be a universal numerable principal π-bundle. Then $E\pi$ is a contractible free right π-space. Moreover, E and B are functors: For any group endomorphism $\alpha: \pi\to\pi$, denote by $E\alpha: E\pi\to E\pi$ and $B\alpha: B\pi\to B\pi$ the induced maps. Equip $E\pi$ and $B\pi$ with base points $e_0\in E\pi$, $b_0=\omega(e_0)\in B\pi$ fixed by all the maps $E\alpha$ and $B\alpha$, respectively.

Let π be any group and A a $\boldsymbol{Z}(\pi)$-module. Realize the Eilenberg-MacLane space $K(A, n)$, $n\geqq 2$, as a strictly associative H-space with strict unit $0\in K(A, n)$. Since A is a π-module, π acts from the left on $K(A, n)$ by topological group homeomorphisms. As a model for $L(A, n)$, take the total space [5] of the associated fibre bundle

Received February 22, 1938