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NULL 2-TYPE SURFACES IN E^3 ARE CIRCULAR CYLINDERS

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Abstract

In this article we prove that open portions of circular cylinders are the only surfaces in E^3 which are constructed from eigenfunctions of Δ with eigenvalue 0 and an eigenvalue λ $(\neq 0)$.

1. Introduction.

Let M be a connected (not necessary compact) surface in a Euclidean 3-space E^3. Denote by Δ the Laplacian of M associated with the induced metric. Then the position vector x and the mean curvature vector H of M in E^3 satisfy

$$\Delta x = -2H. \tag{1.1}$$

This formula yields the following well-known result: A surface M in E^3 is minimal if and only if all coordinate functions of E^3, restricted to M, are harmonic functions, that is,

$$\Delta x = 0. \tag{1.2}$$

In other words, minimal surfaces are constructed from eigenfunctions of Δ with eigenvalue zero.

According to the famous Douglas and Rado's solutions to the Plateau problem there exist ample examples of minimal surfaces in E^3. The study of minimal surfaces in E^3 has attracted many mathematicians for many years (cf. [3]).

On the other hand, it is easy to see that circular cylinders in E^3 are constructed from harmonic functions and eigenfunctions of Δ with a nonzero eigenvalue, say λ. The position vector of such a surface admits the following simple spectral decomposition:

$$x = x_0 + x_q, \qquad \text{with } \Delta x_0 = 0 \text{ and } \Delta x_q = \lambda x_q, \tag{1.3}$$

for some non-constant maps x_0 and x_q, where λ is a non-zero constant. In the following, we simply call a surface M in a Euclidean space *a surface of null 2-type* if the position vector x of M has the spectral decomposition (1.3).

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