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KODAI MATH. J.
11 (1988), 249-279

MEROMORPHIC FUNCTIONS COVERING CERTAIN FINITE SETS AT THE SAME POINTS

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1. Introduction.

We say that meromorphic functions f and g share a value c provided that $f(z)=c$ if and only if $g(z)=c$ (regardless of multiplicities). When we need to consider their multiplicities, we shall make use of the abbreviation *CM*, following Gundersen [4]. Unless stated otherwise, all functions will be assumed to be nonconstant and meromorphic in the plane. It is also assumed that the reader is familiar with usual notations of Nevanlinna theory of meromorphic functions (see, for example, [5]).

Our main interest in this paper lies in the following question: under what circumstances two different functions share the values? R. Nevanlinna [8, 9] proved that if two functions share five distinct values (possibly including ∞), then they must be identical. The functions $\exp(z)$ and $\exp(-z)$, with the values $0, 1, -1, \infty$, show that here 5 cannot replaced by 4. He has also shown that three or four values are, apart from certain exceptional cases, sufficient to determine a function $f(z)$, if we know in addition the multiplicity of the roots of the equation $f(z)=c$. For entire functions, their relationships have been given in the various specific circumstances. Here we prove some corresponding results for meromorphic functions in the plane.

2. Statement and discussion of results.

Let $\{a_n\}$, $\{b_n\}$ and $\{p_n\}$ be three disjoint sequences with no finite limit points. If there is a meromorphic function f whose zeros, d-points and poles are exactly $\{a_n\}$, $\{b_n\}$ and $\{p_n\}$ respectively, then the given triple $(\{a_n\}, \{b_n\}, \{p_n\})$ is called *the zero-d-pole set*. Here of course d is a nonzero complex number. If further there exists only one such function f, then the triad is said to be *unique*. Unicity in this sense does not hold in general, and we have the following theorem.

THEOREM 1. *Let* $(\{a_n\}, \{b_n\}, \{p_n\})$ *and* $(\{a_n\}, \{c_n\}, \{p_n\})$ *be the zero-one-pole set and the zero-d-pole set of a function N, where* $d\neq 1$. *Then at least one of two given triads is unique, unless N is one of the following forms;*

Received December 21, 1987