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THE q-ANALOGUE OF THE p-ADIC GAMMA FUNCTION

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Introduction.

The p-adic gamma function $\Gamma_p(x)$ was defined and studied by Morita [9] and the p-adic log-gamma function $G_p(x)$ was defined and studied by Diamond [3]. The Morita's gamma function $\Gamma_p(x)$ is defined by

$$\Gamma_p(x)=\lim_{\substack{n\to x\\ \text{in } \boldsymbol{Z}_p}}(-1)^n\prod_{0<j<n}{}^{*} j \qquad \text{for} \quad x\in\boldsymbol{Z}_p,$$

where n runs over positive integers and $\prod^*$ means that indices j divisivle by p are omitted. The Diamond's log-gamma function $G_p(x)$ and $G_p^*(x)$ are defined by

$$G_p(x)=\lim_{n\to\infty}\frac{1}{p^n}\sum_{0\leq j<p^n}(x+j)\{\log(x+j)-1\} \qquad \text{for} \quad x\in\boldsymbol{C}_p-\boldsymbol{Z}_p$$

and

$$G_p^*(x)=\lim_{n\to\infty}\frac{1}{p^n}\sum_{0\leq j<p^n}{}^{*}(x+j)\{\log(x+j)-1\} \qquad \text{for} \quad x\in\boldsymbol{C}_p-\boldsymbol{Z}_p^*,$$

where log is the Iwasawa p-adic logarithm [5], $\boldsymbol{C}_p$ denotes the completion of the algebraic closure of the p-adic number field $\boldsymbol{Q}_p$ and $\sum^*$ means that indices j divisible by p are omitted in the summation.

Then $G_p(x)$ and $G_p^*(x)$ have the following two connections with $\Gamma_p(x)$.

THEOREM (Diamond [3], Ferrero-Greenberg [4]).

$$(1) \qquad \log\Gamma_p(x)=G_p^*(x) \qquad \textit{for} \quad x\in p\boldsymbol{Z}_p.$$

$$(2) \qquad \log\Gamma_p(x)=\sum_{\substack{0\leq i<p\\ x+i\in\boldsymbol{Z}_p^*}} G_p\left(\frac{x+i}{p}\right) \qquad \textit{for} \quad x\in\boldsymbol{Z}_p.$$

A generalized p-adic gamma function $\Gamma_{p,q}(x)$, depending on a parameter $q\in\boldsymbol{C}_p$ with $|q-1|_p<1$ and $q\neq1$, was defined and studied by Koblitz [7], [8]. We recall that the Koblitz' function $\Gamma_{p,q}(x)$ is defined by

$$\Gamma_{p,q}(x)=\lim_{\substack{n\to x\\ \text{in } \boldsymbol{Z}_p}}(-1)^n\prod_{0<j<n}{}^{*}\frac{1-q^j}{1-q} \qquad \text{for} \quad x\in\boldsymbol{Z}_p,$$

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