

ASYMPTOTIC BEHAVIOR OF ALMOST-ORBITS OF SEMIGROUPS OF LIPSCHITZIAN MAPPINGS IN BANACH SPACES

BY WATARU TAKAHASHI AND PEI-JUN ZHANG

Abstract

Let C be a nonempty closed convex subset of a uniformly convex Banach space E , G a right reversible semitopological semigroup and $S = \{S(t) : t \in G\}$ a continuous representation of G as Lipschitzian self-mappings on C . We consider the asymptotic behavior of an almost-orbit $\{u(t) : t \in G\}$ of $S = \{S(t) : t \in G\}$. We show that if E has a Fréchet differentiable norm and if $\limsup_t k_t \leq 1$, then the closed convex set

$$\bigcap_{s \in G} \overline{co} \{u(t) : t \geq s\} \cap F(S)$$

consists of at most one point, where k_t is the Lipschitzian constant of $S(t)$. This result is applied to study the problem of weak convergence of the net $\{u(t) : t \in G\}$.

1. Introduction.

Let C be a nonempty closed convex subset of a real Banach space E and let T be a mapping of C into itself. T is said to be a Lipschitzian mapping if for each $n \geq 1$ there exists a positive real number k_n such that

$$|T^n x - T^n y| \leq k_n |x - y|$$

for all $x, y \in C$. A Lipschitzian mapping is said to be nonexpansive if $k_n = 1$ for all $n \geq 1$ and asymptotically nonexpansive if $\lim_n k_n = 1$, respectively. Let

$S = \{S(t) : t \geq 0\}$ be a family of nonexpansive mappings of C into itself such that $S(0) = I$, $S(t+s) = S(t)S(s)$ for all $t, s \in [0, \infty)$ and $S(t)x$ is continuous in $t \in [0, \infty)$ for each $x \in C$. Then S is said to be a nonexpansive semigroup on C . In [1], Bruck introduced the notion of an almost-orbit of a nonexpansive mapping. Miyadera and Kobayashi [11] extended the notion to the case of a nonexpansive semigroup; see also Takahashi and Park [14] for general commutative semigroups. Recently, the authors established the weak convergence of an almost-orbit of a noncommutative Lipschitzian semigroup in a Hilbert space [15]. In this paper, we shall extend the result in [15] to the case of Banach spaces.

Received November 19, 1987