T. OTSUKI
KODAI MATH. J.
11 (1988), 86—128

A CERTAIN PROPERTY OF GEODESICS OF THE FAMILY OF RIEMANNIAN MANIFOLDS O_n^2 (IX)

BY TOMINOSUKE OTSUKI

§ 0. Introduction.

This is exactly a continuation of Part (VIII) ([20]) with the same title written by the present author which proved the following conjecture is true for $4.5 \leqq n \leqq 5$. He aimed at giving the proof of it for $3 \leqq n \leqq 5$ but found that the method by using computors in it was not so effective, when n comes near 3 and we know that the method used until now does not work well for the proof of Conjecture C for $2 \leqq n \leqq 3$ by Lemma 8.1 of Part (III) ([13]). We shall show that this conjecture is also true for $2.4 \leqq n \leqq 4.5$ in the present paper by taking a new way. As the previous ones, we shall use the numerical data obtained by means of computors in the verification. We shall also use the same notation in the previous papers (I)-(VIII).

The period T of any non-trivial solution $x(t)$ of the non-linear differential equation of order 2:

$$\text{(E)} \qquad nx(1-x^2)\frac{d^2x}{dt^2}+\left(\frac{dx}{dt}\right)^2+(1-x^2)(nx^2-1)=0$$

with a constant $n>1$ such that $x^2+x'^2<1$ is given by the integral:

$$\text{(0.1)} \qquad T=\sqrt{nc}\int_{x_0}^{x_1}\frac{dx}{x\sqrt{(n-x)\{x(n-x)^{n-1}-c\}}},$$

where $x_0=n\{\min x(t)\}^2$, $x_1=n\{\max x(t)\}^2$, $0<x_0<1<x_1<n$ and $c=x_0(n-x_0)^{n-1}=x_1(n-x_1)^{n-1}$.

CONJECTURE C. *The period T as a function of $\tau=(x_1-1)/(n-1)$ and n is monotone decreasing with respect to n (>2) for any fixed $\tau(0<\tau<1)$.*

§ 1. Preliminaries.

Setting $T=\Omega(\tau, n)$, we have the formulas

$$\text{(1.1)} \qquad \frac{\partial\Omega(\tau, n)}{\partial n}=-\frac{\sqrt{c}}{2(B-c)n\sqrt{n}}\int_{x_0}^{x_1}\frac{(1-x)\sqrt{x(n-x)^{n-1}-c}}{x^2(n-x)^n}V(x, x_1)dx$$

Received November 10, 1987