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DEFORMATIONS OF SOME ALGEBRAIC SURFACES WITH $q=0$ AND $p_g=1$

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§1. Introduction.

Let M^a be an affine algebraic surface in C^3 defined by $h(w)=1+\sum_{i=1}^{3} w^{A_i}=0$ where A_1, A_2, A_3 are linearly independent non-negative integral vectors. Let Δ be the simplex in R^3 spun by $\vec{0}$, A_1, A_2 and A_3. In [6, 7], Oka showed that M^a has a canonical smooth compactification in a toric variety W of dimension three. Let $A_4, \cdots, A_l$ be the other integral points on Δ and let $h_t(w)=h(w)+\sum_{i=4}^{l} t_i w^{A_i}$. There exists a Zariski open set U^e of C^{l-3} such that the family of affine algebraic surfaces $M_t^a=\{h_t(w)=0\}$ $(t\in U^e)$ has a simultaneous smooth compactification M_t in W $(M_0=M)$. This deformation is called the embedded deformation of M ([7]). Let ν_t be the sheaf of the germs of the holomorphic section of the normal bundle of M_t in W and let Θ_t and Θ_W be the sheaves of the germ of holomorphic vector fields of M_t and W respectively. We have the canonical exact sequence:

$$0 \longrightarrow \Theta_t \longrightarrow \Theta_W|M_t \longrightarrow \nu_t \longrightarrow 0. \tag{1.1}$$

This induces the following long exact sequence:

$$\begin{aligned} 0 &\longrightarrow H^0(M_t, \Theta_t) \longrightarrow H^0(M_t, \Theta_W|M_t) \longrightarrow H^0(M_t, \nu_t) \\ &\xrightarrow{\delta} H^1(M_t, \Theta_t) \longrightarrow H^1(M_t, \Theta_W|M_t) \longrightarrow H^1(M_t, \nu_t) \\ &\longrightarrow \cdots\cdots\cdots. \end{aligned} \tag{1.2}$$

In [7], Oka has studied the infinitesimal displacement map

$$\xi^e : T_t U^e \longrightarrow H^0(M_t, \nu_t), \tag{1.3}$$

and the Kodaira-Spencer map $\delta\circ\xi^e$ where δ is the canonical homomorphism

$$\delta : H^0(M_t, \nu_t) \longrightarrow H^1(M_t, \Theta_t). \tag{1.4}$$

The dimension of Ker δ is at least 3. He gives an example (See §7, [7]) where dim Ker $\delta=3$ and δ is surjective.

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