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ON ANALYTIC MAPS OF PLANE DOMAINS

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Dedicated to Professor Kôtaro Oikawa on his 60th birthday

1. Introduction.

Let D be a finitely connected plane domain, having more than two non-degenerate boundary components. We may assume that D is bounded by analytic curves.

An analytic map $f(z)$ of D is called *boundary preserving*, if every sequence $\{z_\nu\}_{\nu=1}^{\infty}$ of D tending to the boundary ∂D is mapped onto a sequence $\{f(z_\nu)\}_{\nu=1}^{\infty}$ tending to the boundary of the image domain $\Delta=f(D)$. A boundary preserving map $f(z)$ of D covers the image domain Δ finitely many times, N. By making use of circular slit mappings, we shall show a uniqueness theorem of an analytic map $f(z)$.

A boundary preserving map $f(z)$ is extended over the doubled surface $\hat{D}$ and the extended map $\hat{f}(z)$ is an analytic map of the closed Riemann surface $\hat{D}$ onto the doubled surface $\hat{\Delta}$. The Seveli - deFranchis' Theorem [3] states that the number of analytic map from a compact Riemann surface $\hat{D}$ of genus greater than one into a compact Riemann surface $\hat{\Delta}$ of genus greater than one is finite. Recently A. Howard and A. Sommes [2] gave a bound of the number of analytic maps of a compact Reimann surface of genus $g\geqq 2$, which was

$$(2\sqrt{6}(g-1)+1)^{2+2g^2}g^2(g-1)\sqrt{2}^{8(g-1)}+84(g-1).$$

If the connectivity of D in $n\geqq 3$, the genus of D is equal to $n-1$. As an application of the uniqueness theorem we obtain a simple bound of the number of boundary preserving maps of D into domain of connectivity $\geqq 3$

$$(n-2)2^{4n-6}.$$

2. Uniqueness problem.

Let D be an n-ply connected plane domain bounded by n analytic curves. Let $f(z)$ be an boundary preserving analytic map of D onto an m-ply connected

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