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REMARKS ON A RESULT OF HAYMAN

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1. Introduction.

In this paper, we use the usual notation of Nevanlinna theory[3].

Suppose that $f(z)=\sum_{n=0}^{\infty} a_n z^{\lambda_n}$ is a transcendental entire function, where $a_n \neq 0$ $(n=0, 1, 2, \cdots)$ and $\{\lambda_n\}$ is arranged in increasing order. Also let $g(z)$ be an arbitrary entire function growing slowly compared with the function $f(z)$, i.e., $T(r, g)=o\{T(r, f)\}$ as $r\to\infty$. Following Hayman[4], if $f(z)$ has finite order, we define

$$\delta_S(g(z), f)=1-\underline{\lim}_{r\to\infty}\frac{N\left(r, \frac{1}{f-g(z)}\right)}{T(r, f)}.$$

If $f(z)$ has infinite order, let E be any set in $(1, \infty)$ having finite length. We define

$$\delta_S(g(z), f)=1-\sup_E \underline{\lim}_{r\to\infty, r\notin E}\frac{N\left(r, \frac{1}{f-g(z)}\right)}{T(r, f)}=\inf_E \overline{\lim}_{r\to\infty, r\notin E}\frac{m\left(r, \frac{1}{f-g(z)}\right)}{T(r, f)}.$$

Obviously,

$$\delta(g(z), f)=1-\overline{\lim}_{r\to\infty}\frac{N\left(r, \frac{1}{f-g(z)}\right)}{T(r, f)}\leqq \delta_S(g(z), f).$$

In particular, when $g(z)\equiv a$ (a is a constant) we get the definition of $\delta_S(a, f)$ defined by Hayman[4].

Under the above definitions, Hayman[4] proved

THEOREM A. *Let d_n be the highest common factor of all the numbers $\lambda_{m+1}-\lambda_m$ for $m\geqq n$ and suppose that*

$$d_n \longrightarrow \infty \quad \text{as} \quad n\to\infty.$$

Then $\delta_S(a, f)=0$ for every finite complex number a.

With the hypotheses of Theorem A, we proved in [2] $\Theta_S(g(z), f)\leqq 1/2$ for every function $g(z)$ satisfying $T(r, g)=o\{T(r, f)\}$. Now we further prove

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