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ASYMPTOTIC BEHAVIOR OF PERIODIC SOLUTIONS IN BANACH SPACE

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1. Introduction.

We consider the following problem:

$$\frac{du(t)}{dt}+Au(t)\ni f(t), \quad t\in(0, \infty),$$
$$u(0)=x, \tag{1}$$

where A is an m-accretive operator in Banach space X and $f\in L^1_{loc}(0, \infty; X)$ is T-periodic. Let $\{C_t\}_{t\geqq 0}$ be a nonempty closed convex subset of a Banach space and let $U=\{U(t, s): 0\geqq s\geqq t\}$ be a nonexpansive operator constrained in $\{C_t\}$, i.e., U is a family of mappings $U(t, s): C_s\to C_t$ such that

$$U(t, s)U(s, r)=U(t, r), \quad U(r, r)=I,$$
$$|U(t, s)x-U(t, s)y|\leqq|x-y|$$

for all $0\leqq r\leqq s\leqq t$ and $x, y\in C_s$. Such an evolution operator U is said to be T-periodic ($T>0$) if

$$C_{t+T}=C_t \quad\text{and}\quad U(t+T, s+T)=U(t, s)$$

for all $0\leqq s\leqq t$. Then, a function $u: [0, \infty)\to X$ is an almost semitrajectory of U if

$$\lim_{s\to\infty}\sup_{t\geqq s}|u(t)-U(t, s)u(s)|=0.$$

In what follows, let $U=\{U(t, s): 0\leqq s\leqq t\}$ be a T-periodic nonexpansive evolution operator constrained in $\{C_t\}$ and we take $u(t)=U(t, 0)u(0)$ for $t\geqq 0$. We shall denote $u(nT+t)$ by $u_n(t)$.

If $F(U_t)=\{x: U(T+t, t)x=x \text{ for } 0\leqq t\leqq T\}$ is nonempty, then we can take $z\in F(U_t)$, and we see that

$$\lim_{n\to\infty}|u_n(t)-z|=\rho(t)$$

exists. It is well known [1] that (1) has a unique integral solution $U(t; s, x)$

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