K. MAEHARA
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# THE MORDELL-BOMBIERI-NOGUCHI CONJECTURE OVER FUNCTION FIELDS

By Kazuhisa Maehara

§1. **Introduction.** G. Faltings recently proved the Mordell conjecture [F]. The author learned from J. Noguchi that E. Bombieri made the following conjecture generalizing the conjecture above (cf. also [L]):

*The set of rational points of any projective variety of general type over an algebraic number field is not Zariski dense.*

Noguchi ([N1], [N2]) has obtained some results over function fields which are analogues of the Bombieri conjecture.

Conjecture A (Noguchi). *Let $f: X\to S$ be a proper surjective map between non-singular projective varieties over the complex number field. Let $\sigma_\lambda$ denote the rational sections of $f$. Assume that a general fibre $X_s$ of $f$ is a variety of general type and that the union of $S_\lambda=\sigma_\lambda(S)$ is Zariski dense in $X$. Then $X$ is birationally trivial, i.e., there exists a projective variety $X_0$ such that $X$ is birational to $X_0\times S$.*

We pose the following conjecture, which implies Conjecture A.

Conjecture B. *Let $X$ and $S$ be non-singular projective varieties. Then there exists an ample divisor $D$ on $S$ such that for any birational embedding $j_\lambda$: $S\to X$ we have $\mathcal{O}(j_\lambda^* K_X)\subset\mathcal{O}(D)$.*

Note that when $X$ is the minimal model of a surface with $\kappa(X)\geqq 0$, Miyaoka and Umezu proved Conjecture B([MU]). We shall prove Conjecture A with additional assumptions:

Main Theorem. *Let $f: X\to S$ be a proper surjective map between non-singular projective varieties over the complex number field. Let $\sigma_\lambda$ denote the rational sections of $f$. Assume that a general fibre $X_s$ of $f$ is a variety of general type and the union of $S_\lambda=\sigma_\lambda(S)$ is Zariski dense in $X$. Let $P$ denote the projective bundle $p: P(\Omega_X^s)\to X$, where $s=\dim S$. Furthermore suppose that $\mathcal{O}(\alpha)\otimes p^*\mathcal{O}(-K_X)$ is $f\circ p$-nef for some $\alpha>0$. Then $X$ is birationally trivial over $S$.*

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