§ 1. Introduction.

Let $C$ be a bounded closed convex subset of a Banach space $E$ and let $T$ be a nonexpansive mapping from $C$ into itself. Browder [2] and Göhde [10] showed that if $E$ is uniformly convex then $T$ has a fixed point, while Kirk [13] proved that if $E$ is reflexive and if $C$ has normal structure then $T$ has a fixed point. On the other hand, Goebel [7] defined the characteristic $\varepsilon_0$ of convexity of $E$ and showed that $E$ is uniformly convex if and only if $\varepsilon_0 = 0$, if $\varepsilon_0 < 1$ then $E$ has normal structure and if $\varepsilon_0 < 2$ then $E$ is reflexive. Also, Bynum [3] defined the normal structure coefficient $N(E)$ of $E$, and then Maluta [17] and Bae [1] proved that if $N(E)^{-1} < 1$ then $E$ is reflexive and has normal structure. Using these coefficients, Goebel and Kirk [8], Goebel, Kirk and Thele [9] and Casini and Maluta [4] proved the fixed point theorems for uniformly $k$-lipschitzian mappings. (For the results on Hilbert space, see [5], [12], [14].) But it seems natural to define these coefficients for a convex set, since for any Banach space $E$, a nonexpansive mapping has a fixed point if $C$ is weakly compact and has normal structure.

In this paper, we introduce the modulus $\delta(C, \varepsilon)$ of convexity, the characteristic $\varepsilon_0(C)$ of convexity and the constant $\tilde{N}(C)$ of uniformity of normal structure for a convex subset $C$ of a Banach space and prove some results similar to [3], [7], [11], [17]. For example, we show that if $\tilde{N}(C) < 1$ then $C$ is boundedly weakly compact. Further, by using these coefficients, we prove three fixed point theorems. All of these proofs are given by explicitly constructing a sequence which converges to a fixed point. We first show a fixed point theorem for nonexpansive semigroups. Secondly, we obtain a fixed point theorem for uniformly $k$-lipschitzian semigroups on $C$ under $k < \gamma$, where $\gamma$ is determined by the modulus of convexity of $C$. Also, using our results, we evaluate $\gamma$ as $1 < \gamma \leq 1 + (1 - \varepsilon_0(C))/2$. Finally, we prove that Casini and Maluta’s result [4] is valid under more general semigroups.

§ 2. Preliminaries.

Let $E$ be a real Banach space and let $B$ be a bounded subset of $E$. For a